Problem 1, 34-25

a) Use $\lambda f = c$, \[ f = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{3.0 \text{ m}} = 1 \times 10^8 \text{ s}^{-1} \]

b) if $E$ is along y axis, $B$ points along z axis
   So that $E \times B$ points in $x$ direction

\[ B_0 = \frac{E_0}{c} = \frac{300 \text{ V/m}}{3 \times 10^8 \text{ m/s}} = 1 \times 10^{-4} \text{ T} \]

c) $\omega = 2\pi f = 6.3 \times 10^8 \text{ s}^{-1}$
   \[ k = 2\pi / \lambda = 2.1 \text{ m}^{-1} \]

d) $I = \frac{1}{2} \frac{E_0 B_0}{\mu_0} = \frac{1}{2} \frac{300 \text{ V/m} \times 1 \times 10^{-4} \text{ T}}{4\pi \times 10^{-7} \text{ TmA}^{-1}} = 119 \text{ Wm}^{-2}$

e) the rate of momentum transfer is \[ \frac{dp}{dt} = \frac{IA}{c} \]

\[ \frac{dp}{dt} = \frac{119 \text{ Wm}^{-2} \times 2 \text{ m}^2}{3 \times 10^8 \text{ m/s}} = 8 \times 10^{-7} \text{ N} \]

The radiation pressure is force/area, \[ F = \frac{dp}{dt} \]
So \[ P = \frac{8 \times 10^{-7} \text{ N}}{2 \text{ m}^2} = 4 \times 10^{-7} \text{ Nm}^{-2} \]

Problem 2, 34-27

This problem is worded somewhat poorly. It should have said "is equal to the energy density in the total EM wave on the incident side of the surface." This distinction is important because we have to account for the energy density in the incident and reflected waves.
Problem 2 (cont.)

Another assumption of the problem that is not so clearly stated is that whatever light is not reflected from the surface is absorbed.

So, assume a fraction $f$ is reflected. That reflected light contributes $\frac{2T_o f}{c}$ to the radiation pressure on the surface while the absorbed light contributes $(1-f)T_o$.

So, the total radiation pressure on the surface is 

$$P_r = (1+f)T_o/c$$

Now, the energy density in an electromagnetic wave is given by 

$$U = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2} B^2/\mu_0$$

As we saw in class, this can be re-written using $|B| = E/c$:

$$U = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2} \varepsilon_0 \frac{E^2}{\mu_0 c^2} \quad \text{but} \quad \frac{1}{c^2} = \mu_0 \varepsilon_0$$

So 

$$U = \frac{1}{2} (2 \varepsilon_0 E^2) = \varepsilon_0 E^2 = \varepsilon_0 E(Bc)$$

Once again using $c = \sqrt{\varepsilon_0 \mu_0}$, 

$$U = EB \sqrt{\varepsilon_0 \mu_0} = \frac{EB \sqrt{\varepsilon_0 \mu_0}}{\mu_0}$$

or 

$$U = S/c.$$

So 

$$\langle U \rangle = \langle |S| \rangle c = I/c$$
Problem 2 (cont.)

Now, if we account for both the incident and reflected waves, both contribute to $U$, so we have

$$<U> = \frac{1}{c} E_0 + \frac{i}{c} I_0 = (1+i)I_0/c$$

This is equal to the radiation pressure obtained above.

Problem 3, 34-31

If the particle has density $\rho$ and is a sphere of radius $r$, then it has mass $\frac{4}{3}\pi r^3 \rho$. The gravitational force on it from a body of mass $M$ is

$$F_g = G M \left( \frac{\frac{4}{3}\pi r^3 \rho}{R^2} \right)$$

Now, the radiation pressure on the particle will depend on distance from the sun because the intensity will decrease as $1/R^2$, i.e. if $I$ is total power output of the sun, $I = P/4\pi R^2$

Now we can't simply multiply $Pr = I \hat{r}$ times surface area of the sphere because the radiation pressure on the wave is not normal to the surface of the sphere everywhere over its surface. In principle we need to evaluate $Pr \cdot \hat{n}$ and integrate over the surface of the sphere.
Problem 3 (cont.)

However, all radiation passing through a circle of radius \( r \) will be absorbed by the sphere so regardless of the variation of \( F_r \) over the sphere’s surface we calculate the total radiation force on it by calculating

\[
F_r \times \pi r^2 = F_r = \frac{P}{4\pi R^2} \pi r^2 = \frac{P}{4\pi} \frac{r^2}{R^2}
\]

Since the EM waves will propagate radially from the sun, \( F_r \) will point \( r \) radially, thus, \( F_r \) points in opposite direction from \( F_g \).

Since both forces decrease as \( r^2 \), the determination of whether \( F_r > F_g \) depends only on \( r \), the radius of the sphere.

Then, if \( F_r = F_g \), the two forces exactly balance,

this happens when: \[
\frac{4}{3} \pi GM \frac{r_c^3 P}{R^2} = \frac{P}{4\pi} \frac{r^2}{R^2}
\]

or when \( r_c = \frac{3P}{16\pi GM} \).

If \( r < r_c \), \( F_r > F_g \) and the particle is blown away from the sun by the radiation.
Problem 4, 34-37

The angle of the first filter $\theta_1$ with respect to the incoming wave is 70°. The angle of the second filter with respect to the first, $\theta_2$, is 20° (90° - 70°).

The intensity passing the first filter is given by

$$I_1 = I_0 \cos^2(\theta_1) = 4.3 \text{ Wm}^{-2} \times \cos^2(70°) = 5.03 \text{ Wm}^{-2}$$

The intensity passing the second filter is $I_2 = I_1 \cos^2(\theta_2)$

$$I_2 = 5.03 \text{ Wm}^{-2} \times \cos^2(20°) = 4.4 \text{ Wm}^{-2}$$

Problem 5, 34-40

The total electric field in the light reflected from the sand is $E = E_H \hat{\imath} + E_V \hat{\jmath}$. Amplitudes of $H \& V$ components

Now, $I_0 = \frac{1}{2} \frac{E_0^2}{c\mu_0} = \frac{1}{2} \frac{1}{c\mu_0} \left( E_{H_0}^2 + E_{V_0}^2 \right)$

a) Now, if the horizontal component is removed the $I = \frac{1}{2} \frac{1}{c\mu_0} E_{V_0}^2$. So $I/I_0 = \frac{E_{V_0}^2}{E_{H_0}^2 + E_{V_0}^2}$

So $I/I_0 = \frac{1}{2.3^2 + 1} = 0.16$

b) Now $I/I_0 = \frac{E_{H_0}^2}{E_{H_0}^2 + E_{V_0}^2} = \frac{2.3^2}{2.3^2 + 1} = 0.84$
Problem 6, 34-47

Consider a ray that touches the very top of the pole:

\[ h_1 = 0.5 \text{ m} \]

\[ h_2 = 2 \text{ m} - 0.5 \text{ m} = 1.5 \text{ m} \]

From simple geometry \[ x = h_1 \tan \theta_1 = 0.5 \text{ m} \times \tan 35^\circ = 0.35 \text{ m} \]

Now \[ L - x = h_2 \tan \theta_2 \] so \[ L = x + h_2 \tan \theta_2 \]

Now \[ n_{H_2O} \sin \theta_2 = n_{air} \sin \theta_1 \] so \[ \sin \theta_2 = \frac{1}{n_{H_2O}} \sin \theta_1 \]

So \[ \sin \theta_2 = \frac{1}{1.33} \sin(35^\circ) = 0.43 \] so \[ \theta_2 = 25.5^\circ \]

So \[ L = 0.35 \text{ m} + 1.5 \text{ m} \times \tan(25.5^\circ) = 1.07 \text{ m} \]

Problem 7, 34-49

Use Snell's law:

\[ n \sin \theta = n' \sin \theta' \]

This applies at both sides so incident and outgoing rays are parallel.
Problem 7 (cont.)

We can use the length of path of the ray in the glass to relate \( x, t \):
\[
\begin{align*}
    t &= 2l \cos \theta' \\
    x &= l \sin (\theta - \theta')
\end{align*}
\]

So \( x = \frac{t}{\cos \theta'} \left( \sin \theta' - \cos \theta \sin \theta' \right) \)

or \( x = t (\sin \theta - \cos \theta \tan \theta') \)

Now because \( n' > n \), \( \sin \theta' < \sin \theta \rightarrow \theta' < \theta \)

So if \( \theta \) is small, \( \theta' \) is smaller. Can use small-angle approximations: \( \sin \theta \approx \theta \) \( \cos \theta \approx 1 \)

for both.

\( \tan \theta \approx \theta \)

So \( x = t (\theta - \theta') \quad \theta' = \frac{n'}{n} \theta \quad n = 1 \quad n' = n_g \) (glass)

So \( x = t \theta (1 - \frac{1}{n_g}) = t \theta (\frac{n_g - 1}{n_g}) \)

Problem 8 (34 - 50)

a) If the incident ray is normal to the surface of the water, it remains normal to the surface in the water. Thus, it strikes the mirror at a 45° angle and is reflected horizontally. When the light reflects from the second mirror it travels vertically passing normal to the surface and emerges parallel to the incident light.
Problem 8 (cont.)

b) Now, if the light enters the water at angle $\theta$ we need more careful analysis. Start by considering a ray already in the water at angle $\theta$ with respect to the vertical.

Now $\theta' = 45^\circ - \theta$ & $\theta'' = 180^\circ - 90^\circ - \theta' = 45^\circ + \theta$

Since the normal to the second mirror is rotated $+45^\circ$ from the vertical, the angle of the final outgoing ray is $-\theta$ from the vertical which is the same angle of the initial ray.

Now since both rays undergo refraction $\theta_{air} = \theta_{H_2O} \frac{n_{H_2O}}{n_{air}}$

at the surface, they are also parallel in air.