Problem 1, 33-35p

a) \( E = E_m \sin (\omega t - \frac{\pi}{4}) \) \( \omega d = 350 \text{ s}^{-1} \)

\( E \) will be maximum when \( \sin (\omega t - \frac{\pi}{4}) = 1 \) or
when \( \omega t - \frac{\pi}{4} = \frac{\pi}{2}, \frac{5\pi}{2}, \ldots \)

The earliest time is \( \omega t = \frac{3\pi}{4} \), or \( t = \frac{3\pi}{4\omega d} \)

This gives \( t = 6.7 \times 10^{-3} \text{ s} \)

b) \( i(t) = I \sin (\omega t - \frac{3\pi}{4}) \)

Maximum when \( \omega t = \frac{7\pi}{4} \)

So \( t = \frac{7\pi}{4\omega d} = 1.1 \times 10^{-2} \text{ s} \)

c) if we write \( E = E_m \sin (\omega t - \phi_0) \)

\( i = I \sin (\omega t - \phi_0 - \phi) \)

Where \( \phi \) is the phase difference between the emf & current, \( \phi = \frac{\pi}{2} \)

This means the circuit element is an inductor.

d) \( I = 620 \text{ mA} \) if we only have the inductor then the current amplitude is related to the emf amplitude by \( E_m = I \times X_L \)

With \( X_L = \omega d \) \( \text{So } L = \frac{E_m}{I \omega d} \)

So \( L = \frac{300 \text{ V}}{0.620 \text{ A \times 350 s}^{-1}} = 0.138 \text{ H} \)
Problem 2, 33-37E

With the capacitor removed we have

\[ Z = \sqrt{R^2 + X_L^2} \]
\[ \phi = \tan^{-1}\left(\frac{X_L}{R}\right) \]

Now, in the problem \( X_L = 86.7 \Omega, \ R = 200 \Omega \)

*Note* the book's solutions use the wrong value for \( R \)
160 \( \Omega \) instead of 200 so answers in the back
are wrong.

\[ Z = \sqrt{(200 \Omega)^2 + (86.7 \Omega)^2} = 218 \Omega \]

then \( I = \frac{E_m}{Z} = \frac{36.0 \text{ V}}{218 \Omega} = 0.165 \text{ A} \)

finally \( \phi = \tan^{-1}\left(\frac{86.7}{200}\right) = \tan^{-1}(0.43) = 23.4^\circ \)

Now \( i \) the current is written \( i = I \sin(\omega t - \phi) \)
because \( \phi \) is positive the current is at a smaller
angle than \( E \) on a phasor diagram. I prefer
to write these with \( E_m \) on the x axis:

\[ \begin{align*}
E_m & \quad \downarrow \quad \phi \\
\phi & \quad \uparrow \quad \downarrow \\
V_L & \quad \downarrow \\
V_R & \quad \downarrow
\end{align*} \]

Since \( V_R \) is in phase with \( i \),
\( V_R \) is \( \phi \) degrees below \( E \),
\( V_L \) is always 90° greater
angle than \( V_R \)
Problem 3, 41P

The answer is "yes" as the specified example demonstrates. At resonance \( W = \frac{1}{\sqrt{LC}} \)

Then, \( X_L = \omega L = \sqrt{\frac{1}{LC}} \) and \( Z = R \)

So \( I = \frac{E_m}{R} \), \( V_L = I X_L = \frac{E_m \sqrt{C}}{R} \)

So if \( \sqrt{\frac{1}{LC}} > R \), then the amplitude across the inductor will be greater than the amplitude of the voltage source (emf).

In the stated example \( \sqrt{\frac{1}{LC}} = \left( \frac{1.0 H}{1 \times 10^{-6} F} \right)^{\frac{1}{2}} = 1000 \Omega \)

\( R \) is only 10 \( \Omega \) so \( V_L \) is a factor of 100 greater than \( E_m \).

Problem 4, 33-45P

a) The maximum current occurs when \( Z \) is minimum, i.e. when \( Z = \sqrt{R^2 + (X_L - X_C)^2} = R \)

this happens when \( X_L = X_C \) or \( \omega L = \frac{1}{\omega C} \Rightarrow W = \frac{1}{\sqrt{LC}} \)

So \( W_R = \left( 1.0 H \times 2 \times 10^{-5} F \right)^{\frac{1}{2}} = 224 \) s⁻¹

b) \( Z = R \) at \( W = W_R \) So \( I = \frac{30 V}{5 \Omega} = 6.00 \) A
Problem 4, Cont.

The current will reach its half-maximum value when \( Z = 2R \) or when \( R^2 + (X_L - X_C)^2 = 4R^2 \).

So we want \( (WL - \frac{1}{WL})^2 = 3R^2 \).

Factor out \( WL \) from \( (WL)^2 \) and multiply both sides by \( C^2 \) so we get:

\[
W^2L^2C^2 \left( 1 - \frac{1}{W^2LC} \right)^2 = 3R^2C^2
\]

Now, \( WR = \frac{1}{\sqrt{LC}} \).

So we can re-write the above in terms of \( W/WR \):

\[
\frac{W^2}{WR^4} \left( 1 - \frac{WR^2}{W^2} \right)^2 = 3R^2C^2
\]

Multiply by \( WR^2 \) and expand \( ( )^2 \):

\[
\frac{W^2}{WR^2} \left( 1 - 2 \frac{WR^2}{W^2} + \frac{WR^4}{W^4} \right) = 3R^2C^2WR^2
\]

\[
= \frac{W^2}{WR^2} - 2 + \frac{WR^2}{W^2}
\]

Rewrite \( \left( \frac{W}{WR} \right)^2 - \left( 2 + 3R^2C^2WR^2 \right) + \left( \frac{WR}{W} \right)^2 = 0 \).

Multiply through by \( W^2/WR^2 \) to make quadratic eqn.

\[
(W/WR)^4 - (2 + 3R^2C^2WR^2)(W/WR)^2 + 1 = 0
\]
Problem 4 (cont.)

So, we can use the quadratic equation to obtain

\[
\left(\frac{w}{wr}\right)^2 + \left(\frac{w}{wr}\right) = \frac{2+3R^2C^2wr^2}{2} \pm \sqrt{(2+3R^2C^2wr^2)^2-4} \times \frac{1}{2}
\]

or

\[
\left(\frac{w}{wr}\right)^2 = 1 + \frac{3}{2}R^2C^2wr^2 \pm \sqrt{9R^4C^4wr^4+12R^2C^2wr^2} \times \frac{1}{2}
\]

\[
= 1 + \frac{3}{2}R^2C^2wr^2 \pm \frac{RCwr}{2} \sqrt{9R^2C^2wr^2+4} \times 3
\]

Now, \( R = 5\Omega \), \( C = 2 \times 10^{-5} \text{ F} \) \( wr = 224 \) \( \text{ so} \)

\[
RCwr = 0.0224
\]

and \( (RCwr)^2 = 5 \times 10^{-4} \)

So, because \( (RCwr)^2 \) is so small, we can ignore those terms compared to terms of order 1
then we get

\[
\left(\frac{w}{wr}\right)^2 = 1 \pm RCwr \sqrt{3} = 1 \pm 0.039
\]

or \( w = wr \sqrt{1 \pm 0.039} \)

\[
w_+ = 224 (1 + 0.039)^{1/2} = 228 \text{ s}^{-1}
\]

\[
w_- = 224 (1 - 0.039)^{1/2} = 220 \text{ s}^{-1}
\]
Problem 5, 33-54E

This problem is straightforward starting from

\[ P_{avg} = E_{rms} \cdot I_{rms} \cdot \cos \phi \]

Now \[ \tan \phi = \frac{X_L - X_C}{R} \] and \[ (1 + \tan^2 \phi)R^2 = R^2 + (X_L - X_C)^2 = Z^2 \]

Using the trigonometric identity \[ 1 + \tan^2 \phi = \frac{1}{\cos^2 \phi} \]

then \[ \frac{R^2}{\cos^2 \phi} = Z^2 \] or \[ \cos \phi = \frac{R}{Z} \]

So \[ P_{avg} = E_{rms} \cdot I_{rms} \left( \frac{R}{Z} \right) \] and \[ I_{rms} = \frac{E_{rms}}{Z} \]

Giving \[ P_{avg} = \frac{E_{rms}^2 \cdot R}{Z^2} \]

- For purely resistive circuit \[ Z = R \] and \[ P_{avg} = \frac{E_{rms}^2}{R} \]

which is what we showed in class.

- For RLC Circuit at resonance, \[ Z = R \] and we expect it to behave like a resistive circuit as above.

- For purely inductive or capacitive loads \[ R = 0 \] and there is no power dissipation as we saw in class.
Problem 3. 579

a) The power factor is \(\cos(42^\circ) = 0.743\)
   Note the minus because we define \(\phi\) from
   \[i = i_0 \sin(wt - \phi)\]

b) The current leads the emf because when \(wt = 0\)
   the emf is zero but the current has already
   increased to \(i_0 \sin(42^\circ)\).

c) \[\tan \phi = \frac{X_L - X_C}{R}\]
   for \(\phi < 0\) but \(\phi > -90^\circ\)

   \[\Rightarrow \tan \phi < 0\]

   Since \(\tan \phi < 0\) \(X_C > X_L\) so the circuit is dominated
   by the capacitive component.

d) No, if the circuit were in a resonance \(\phi = 0\)

e) There must be at least a capacitor and resistor.
   If there were no resistor \(\tan \phi = -\infty \Rightarrow \phi = -90^\circ\)

f) \[P_{ac} = E_{rms} I_{rms} \cos \phi = \frac{1}{2} E_{rms} I_{rms} \cos \phi\]
   \[= 0.5 \times 75V \times 1.2A \times 0.743 = 33.4W\]

g) We don't need to know \(w\) because everything above is
   determined from the phase. If we had to calculate \(\phi\) from
   \(R, C, L, \ldots\) then we'd need \(w\).
Problem 2. 63E

We didn't discuss transformers in class but they are a trivial application of what we've learned.

As the book demonstrates, from a simple application of inductance the relationship between the voltages across the "primary" and "secondary" terminals is given by

\[ V_s = N_s \left( \frac{V_p}{N_p} \right) \]

So \( V_s = \frac{120}{500} \left( \frac{10}{500} \right) = 2.4 \text{ V} \)

b) we can easily find \( I_s \), the amplitude of the current oscillation in the secondary coil, \( I_s = \frac{V_s}{R} \)

So \( I_s = \frac{2.4 \text{ V}}{152} = 0.16 \text{ A} \)

The book also shows that by conservation of energy, \( I_s V_s = I_p V_p \)

So \( I_p = I_s \frac{V_s}{V_p} = I_s \frac{N_s}{N_p} \)

So \( I_p = 0.16 \text{ A} \times \frac{10}{500} = 3.2 \times 10^{-3} \text{ A} \).
Problem 8, 34-9E

\[ E = (2.0 \ \text{Nm}^{-1}) \vec{r} \cos \left( \pi \times 10^5 \text{s}^{-1} (t - x/v) \right) \]

This is a wave propagating in the +x direction. Now, \( B \) must be \( \perp \) to \( E \) with \( E \times B \) pointing in the +x direction since \( \vec{S} = \frac{E \times B}{\mu_0} \).

That means the amplitude vector for the magnetic component must point in the -y or -z direction.

We also know that \( |\vec{E}_0| = |\vec{E}|/c \) so
\[ |\vec{B}_0| = \frac{2.0 \ \text{Nm}^{-1}}{3 \times 10^8 \ \text{m/s}^2} = 6.7 \times 10^{-9} \ \text{T} \]

So \( \vec{B} = -6.7 \times 10^{-9} \ \text{T} \ \vec{S} \cos \left( \pi \times 10^5 \text{s}^{-1} (t - x/v) \right) \)

Problem 9, 15E

a) The maximum magnitude of \( E \) is just the amplitude of the electric component, \( |\vec{E}_0| \). As above the amplitude of the magnetic component is \( |\vec{B}_0| = |\vec{E}_0|/c \)

So \( |\vec{B}_0| = \frac{5.0 \ \text{V/m}}{3 \times 10^8 \ \text{m/s}^2} = 1.67 \times 10^{-8} \ \text{T} \)
Problem 9 (cont.)

b) \( I = \langle S \rangle = \frac{1}{2} \frac{E_o B_o}{M_o} = \frac{1}{2} \frac{5.0 \text{Vm}^{-1} \times 1.67 \times 10^{-8} T}{4 \pi \times 10^{-7} \text{TmA}^{-1}} \)

\[ = 3.3 \times 10^{-2} \text{VA} \text{m}^{-2} \]

\[ = 3.3 \times 10^{-2} \text{W} \text{m}^{-2} \]

Problem 10, 16p

\[ I = \frac{1}{2} \frac{E_o B_o}{M_o} = \frac{1}{2} \frac{E_o^2}{M_o C} \Rightarrow E_o = \sqrt{2 I M_o C} \]

\[ E_o = \left( 2 \times 1.4 \times 10^3 \text{W} \text{m}^{-2} \times 4 \pi \times 10^{-7} \text{Tm}^{-1} \times 3 \times 10^8 \text{ms}^{-1} \right)^{\frac{1}{2}} \]

\[ = 1.03 \times 10^3 \text{Vm}^{-1} \]

\[ B_o = \frac{E_o C}{3 \times 10^8 \text{ms}^{-1}} = 3.42 \times 10^{-6} \text{T} \]

Problem 11, 34-17p

Again \( B_o = \frac{E_o C}{3 \times 10^8 \text{ms}^{-1}} = 2.0 \text{Vm}^{-1} \)

\[ \text{I} = \frac{1}{2} \frac{B_o E_o}{M_o} = \frac{1}{2} \left( \frac{2.0 \text{Vm}^{-1} \times 6.7 \times 10^{-9} \text{T}}{4 \pi \times 10^{-7} \text{TmA}^{-1}} \right) = 5.3 \times 10^{-3} \text{W} \text{m}^{-2} \]

The total power is given by \( P = \text{I} \text{A} = 4 \pi r^2 \text{I} \)

\[ P = 5.3 \times 10^{-3} \text{W} \text{m}^{-2} \times 4 \pi \left( 10 \text{m} \right)^2 = 6.7 \text{W} \]