Problem 1, 29-13

The Hall electric field can be calculated by requiring the magnetic force to equal the force due to the Hall field to cancel,

\[ qE = q \vec{u}_d \times \vec{B} \rightarrow E = \vec{u}_d \times \vec{B} \]

Since for this problem \( \vec{u}_d \) is \( \vec{B} \) are perpendicular, \( E = \vec{u}_d \cdot \vec{B} \)

a) The drift velocity can be related back to the current density, \( \vec{u}_d = \frac{j}{ne} \) and we have \( j = Ec \rho \)

So \( E = \frac{Ec \rho}{j} \rightarrow E/E_c = \frac{\rho}{ne} \)

b) \( |\vec{B}| = 0.65 \text{T} \)

\[ \begin{align*}
\rho & = 1.69 \times 10^{-8} \text{S/m} \\
n & = 8.47 \times 10^{28} \text{m}^{-3}
\end{align*} \]

\[ E/E_c = \frac{0.65T}{1.6 \times 10^{-10} \text{C} \times 1.69 \times 10^{-8} \text{m} \times 8.47 \times 10^{28} \text{m}^{-3}} = 2.84 \times 10^{-3} \]

\[ \Rightarrow \text{Use } 1 \Omega = 1 \text{V} \cdot \text{A}^{-1} = 1 \text{V} \cdot \text{s}^{-1} = 1 \text{NC}^{-1} \text{m} \]

then \( \frac{T \text{m}^2}{C \Omega} = \frac{T \text{m}}{NC^{-1} \text{S}} \) but from \( \vec{F} = q \vec{u}_d \times \vec{B} \)

\[ 1 \text{N} = 1 \text{C} \text{m} \text{s}^{-1} \text{ T} \]

\[ \Rightarrow 1 \text{ (no units)} \]

So \( E/E_c = 2.84 \times 10^{-3} \)
Problem 2, 29-38

Since B is constant,
\[ \mathbf{F} = i \mathbf{E} \times \mathbf{B} \]
\[ |\mathbf{F}| = i \mathbf{E} \cdot \mathbf{B} \]

then \[ i = |\mathbf{F}| / \mathbf{B} = \frac{10^4 N}{3 m \times 10^{-5} T} = 3.3 \times 10^8 \text{ NT}^{-1}m^2 \]
= \[3.3 \times 10^8 \text{A} \]

\footnotesize{look back at problem 
\#1, \text{NT}^{-1}m^2 = \text{CS}^2}

b) \[ P = i^2 R, \quad P/R = i^2 = 1.1 \times 10^{17} \text{W} \Omega^{-1} \]

c) Note that the power delivered to the train if it were moving at 180 km hr^{-1} (~110 mph) = \[ \frac{3 \times 10^5 \text{N} \cdot m}{0.05 \text{ km/s}} \]

Would be \[ P = FU = \frac{10^4 N \times 50 \text{ ms}^{-1}}{0.05 \text{ km/s}} \]
or \[ P = 5 \times 10^5 \text{W} \]

Thus, for any reasonable resistance of the rails, the power dissipated in the rails will be much greater than that expended in driving the train along the tracks.
Problem 3, 29-42

Start by imagining two side-by-side loops with $\vec{B}$ parallel to the plane of the loops.

Now each loop will have torque $\vec{T} = \vec{I} \times \vec{B}$ with $\vec{I} = i \vec{A} \hat{k}$. If we imagine locking them together mechanically, the two would have combined torque,

$$\vec{T} = (\vec{I}_1 + \vec{I}_2) \times \vec{B} = i (A_1 + A_2) \hat{k} \times \vec{B}$$

for same current $i$ in each loop.

In the middle, the combined current flow is effectively zero because two loops have current flowing in opposite directions.

So we can treat the pair as a single loop with $\vec{I} = i \vec{A} \hat{k} \times \vec{B}$, $A = A_1 + A_2$

Now, what about loops with trapezoidal shapes (angled ends)?

Use trick from above -- add two opposite flowing currents to make our rectangular loop above $+$ triangle.

Now we need to figure out the torque around the axis due to the triangle.
We need to sum up $d\vec{\tau} = \vec{r}xd\vec{F}$ around the triangle,

$$d\vec{\tau} = \vec{r}x (id\vec{x}\hat{z})$$

if we write $dl = dx\hat{i} + dy\hat{j}$, then

$$d\vec{\tau} \cdot \hat{z} = Bdx\hat{z}$$

Then, $d\vec{\tau} = r_1 Bdx\hat{i}$ where $r_1$ is the distance from the axis to point on the triangle where $dx$ is

in our coordinates $(x,y)$ $r_1 = y$ if the $x$ axis is chosen to be at the same location of the rotation axis.

So $\vec{\tau} = \int d\vec{\tau} = \int r_1 Bdx\hat{i} = IB \int dx y \hat{z}$

So $\vec{\tau} = IB \hat{z} A \hat{z}$.

It's important; even though the triangle is displaced from the axis, its torque around the axis is still just $\vec{r}x\hat{z}$.

So the torque from is just $\vec{\tau} = \vec{r}x\hat{z}$ with $\vec{r} = IB \hat{z}$ where $A$ is area of rectangle + triangle.

Now we put this result together with our adjacent loop above.
Each slice can be thought of as a loop that carries current. The cancelling contributions from adjacent loops means that the net current flow is around the circumference of the loop.

Since each slice contributes \( d\mathbf{\mu} = idA \mathbf{\hat{n}} \),
the total magnetic moment of all of the loops is
\[
\mathbf{\mu} = i \mathbf{\hat{n}} \int dA = i \mathbf{\hat{n}} A
\]
This is also the magnetic moment for the single loop by the reasoning above.
Problem 4, 29-43

For a segment of the ring $dl$, the contribution to the total force is $dF = idl \times \hat{B}$.

If we decompose $\hat{B}$ into components $\parallel$ $\perp$ to the plane of the ring,

$$dl \times \hat{B} = dl \times (\hat{B}_\parallel + \hat{B}_\perp) = dl \left( \hat{n}B_\parallel - \hat{\phi}B_\perp \right)$$

So the component of $\hat{B} \perp$ to the ring produces a force that points radially inward everywhere around the ring. When we integrate over the circle these radially inward pointing components cancel. We're then left with only the $dlB_\parallel \hat{n}$ component.

Integrating around circle, $F = \hat{n}B_\parallel i \oint dl = \hat{n}B_\parallel i 2\pi a$

Now, $B_\parallel = lB_1 \sin \theta$ so $F = lB_1 i \sin \theta 2\pi a \hat{n}$

Problem 5, 30-9

Use the Biot-Savart law,

$$d\vec{B} = \frac{M_0}{4\pi} \frac{idl \times \hat{r}}{r^2}$$
Problem 5, Cont.

Along the radial segments $dl$ is (anti) parallel to $\hat{r} \times \hat{r}$ so $d\hat{r} \times \hat{r} = 0$

Along circular segments $dl$ is $\perp$ to $\hat{r}$ so $|d\hat{r} \times \hat{r}| = dl$

So, integrating over circular segments,

$$B_{outer} = \frac{M_0}{4\pi} \int \frac{d\hat{r} \times \hat{r}}{a^2} = \frac{M_0}{4\pi a^2} \int dl \hat{r} = \frac{M_0}{4\pi a^2} \hat{r} \theta a$$

So $B_{outer} = \frac{Mo i}{2a} \hat{r} \left(\frac{\theta}{2\pi}\right)$

Similarly, $B_{inner} = \frac{M_0}{4\pi} \frac{i}{b^2} \int dl (-\hat{r}) = -\frac{M_0}{4\pi b^2} \hat{r} \theta b$

With $B_{inner} = -\frac{Mo i}{2b} \left(\frac{\theta}{2\pi}\right) \hat{r}$

Then, the total field is $B = -\frac{Mo i}{2} \left(\frac{1}{b} - \frac{1}{a}\right) \left(\frac{\theta}{2\pi}\right) \hat{r}$

Or $B = -\frac{Mo i \theta}{4\pi} \left(\frac{1}{b} - \frac{1}{a}\right) \hat{r}$
Problem 6, 30-32

Use Ampere's law, \[ \oint B \cdot d\ell = M_0 i_{\text{enc}} \]

Because of cylindrical symmetry, \( B \) must be constant as function of polar angle \( \theta \) at fixed distance from the center of the conductor.

By analogy with the straight wire we know \( B \) will point tangentially around the conductor, so

\[ \oint B \cdot d\ell = 2\pi r B \]

then \( i_{\text{enc}} = i\pi r^2 / \pi a^2 = i r^2 / a^2 \) for \( r < a \)

\[ = i \quad \text{for} \quad r > a \]

So, \[ 2\pi r B = i(r^2 / a^2) M_0 \quad r < a \Rightarrow B = M_0 i r / a^2 2\pi \]

\[ = M_0 i \quad r > a \Rightarrow B = M_0 i / 2\pi r \]

Note that at \( r = a \), these two expressions are equal.

Plugging in \( M_0 = 4\pi \times 10^{-7} \text{Tm}^{-1} \), \( i = 100 \text{A} \), \( a = 0.02 \text{m} \)

\[ B = 2\times10^{-5} \text{Tm} \times 50 \text{m}^{-1} \times \left( \frac{r}{a} \right) \quad r < a \Rightarrow 1\times10^{-3} \text{T at} \ r = a \]

\[ = 2\times10^{-5} \text{Tm} / r \quad r > a \]
Problem 7, 30-33.

Start by assuming that the field does drop to 0 abruptly.

Apply Ampere's law -- actually calculate \( \oint \mathbf{B} \cdot d\mathbf{s} \) = \( B_0 l \) but Ienc = 0!

Since our assumption would violate Ampere's law, it must be wrong.

Now, suppose the outside field is not zero but \( B_0' \) along the right-hand leg of the loop. If the field is only in the vertical direction then we have

\[ \oint \mathbf{B} \cdot d\mathbf{s} = l (B_0 - B_0') \neq 0 \rightarrow \text{this still violates Ampere's law} \]

So, \( B \cdot d\mathbf{s} \) must be non-zero along the top and bottom sides of the loop \( \Rightarrow \) there must be horizontal components of the field.

Since \( B_0 - B_0' > 0 \) for \( B_0 < B_0' \), the contributions from the top and bottom of the loop must be negative if Ampere's law is to be satisfied.

We would also expect the shape of the field to be symmetric around the mid-plane of the magnet.
Problem 7 (cont.)

The shape of the fringe field shown gives \( \int \vec{B} \cdot d\vec{s} \) on top & bottom allowing
\[
\int \vec{B} \cdot d\vec{s} = 0
\]

Problem 8, 30-39

Biot-Savart law:
\[
d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{x} \times \vec{r}}{r^3} = \frac{\mu_0 i d\vec{x} \times \vec{r}}{4\pi r^2}
\]

I have chosen 3 different points in the sheet of current & drawn their \( \vec{r} \) vectors. \( dl^2 \) is out of the page. I show the directions of \( d\vec{x} \times \vec{r} \) for the three different \( \vec{r} \)'s.

If \( \theta_2 = \theta_3 \), the vertical components of \( d\vec{x} \times \vec{r}_2 \) and \( d\vec{x} \times \vec{r}_3 \) cancel only leaving contributions pointing horizontally and to the left. If the plane (sheet) of current is infinite, then for every point \( \vec{p} \) there will be contributions on left & right that cancel giving only left-pointing \( \vec{B} \) field.
If we look at points below the sheet, the directions of \( \vec{F} \)'s are reversed so \( \vec{B} \) points to the right. Since our analysis doesn't depend on \( 1/l \), we obtain the same result no matter how far we are from the sheet.

Then, since the field lines have to be parallel, the magnitude of \( \vec{B} \) must be constant as a function of position above the sheet or below the sheet.

Now apply Ampere's law:

\[
\int \vec{B} \cdot d\vec{S} = |\vec{B}|l + |\vec{B}|l = 2|B|l
\]

\[i_{enc} = \mu_0 \lambda l\]

So \( 2|\vec{B}|l = \mu_0 \lambda l \)

or \( |\vec{B}| = \frac{\mu_0 \lambda}{2} \)

---

**Problem 9, 30-43**

Plane cut through diameter of the toroid

\( b = a + d \)
Apply Ampere's law to circular path of radius \( r \).

By symmetry, \( B \) will be constant around the toroid.

Then, \( \int \mathbf{B} \cdot d\mathbf{s} = B \cdot 2\pi r \)

Then, for any \( r \) inside the toroid \( i_{enc} = Ni \) where \( N \) is the number of turns.

So \( B = \frac{M_0 i N}{2\pi r} \)

So for a) \( r = a = 0.15 \text{ m} \)

Then \( B = \frac{4\pi \times 10^{-7} \text{ TmA}^{-1} \times 0.8 \text{ A} \times 500}{0.15 \text{ m}} \)

\[ = 5.33 \times 10^{-4} \text{ T} \]

for b) \( r = b = 0.20 \text{ m}, \ B = 2 \times 10^{-7} \text{ TmA}^{-1} \times 0.8 \text{ A} \times 500 \)

\[ = 4.0 \times 10^{-4} \text{ T} \]
Problem 10, 30-46

The magnetic field of a solenoid is \( B = \mu_0 \text{M} \text{O} \text{i} n \).

The radius of orbit for a particle in a constant field is \( r = \frac{p}{qB} \) so \( r = \frac{p}{\mu_0 \text{M} \text{O} \text{i} n} \).

So \( i = \frac{p}{\mu_0 \text{M} \text{O} \text{n}} = \frac{mU}{\mu_0 \text{M} \text{O} \text{n}} = \frac{m}{e} \frac{U}{\mu_0 \text{M} \text{n}} \).

Use \( U = 0.0460 \times 2.998 \times 10^8 \text{m} \text{s}^{-1} = 1.379 \times 10^7 \text{m} \text{s}^{-1} \).

Then \( i = \frac{9.11 \times 10^{-21} \text{kg} \times 1.379 \times 10^7 \text{m} \text{s}^{-1}}{1.6 \times 10^{-19} \text{C} \times 4\pi \times 10^{-7} \text{T} \text{m} \text{A}^{-1} \times 0.023 \text{m} \times 100 \text{m}^{-1}} \).

\[ = 27.17 \text{ A}. \]

Problem 11, 30-55

\( B_1(x) = \frac{\mu_0 i R^2}{2(R^2 + (\frac{S}{2x} + x)^2)^{3/2}} \)

\( B_2(x) = \frac{\mu_0 i R^2}{2(R^2 + (\frac{S}{2x} - x)^2)^{3/2}} \)

\( B = B_1 + B_2 = \frac{\mu_0 i R^2}{2R^3} \left[ \frac{1}{(1 + (\frac{S}{2R} + \frac{x}{R})^2)^{3/2}} + \frac{1}{(1 + (\frac{S}{2R} - \frac{x}{R})^2)^{3/2}} \right] \)
Problem 11 (cont.)

Define \( f_\pm = \left( 1 + (\frac{s}{2R} \pm \frac{x}{R})^2 \right) \)

Then \( B(x) = \frac{M_0 i}{2R} \left( \frac{1}{f_+^{3/2}} + \frac{1}{f_-^{3/2}} \right) \)

So \( \frac{dB}{dx} = \frac{M_0 i}{2R} \left( \frac{-3}{2} \right) \left[ \frac{1}{f_+^{5/2}} \times 2x \left( \frac{s}{2R} + \frac{x}{R} \right) \left( \frac{1}{R} \right) \right. \)

\[ + \frac{1}{f_-^{5/2}} \times 2x \left( \frac{s}{2R} - \frac{x}{R} \right) \left( -\frac{1}{R} \right) \] \]

\[ = -3\frac{M_0 i}{2R^2} \left[ \frac{s}{2R} \left( \frac{1}{f_+^{5/2}} - \frac{1}{f_-^{5/2}} \right) + \frac{x}{R} \left( \frac{1}{f_+^{5/2}} + \frac{1}{f_-^{5/2}} \right) \right] \]

\[ \frac{dB}{dx} \bigg|_{x=0} = -3\frac{M_0 i}{2R^2} \frac{s}{2R} \left( \frac{1}{f_+^{5/2}(0)} - \frac{1}{f_-^{5/2}(0)} \right) = 0 \]

These are equal.