Problem 1, 27-11

First, find the drift velocity. We are given $i = 4$ so we can calculate $J$ since it will be uniform across the conductor, $J = i/A$.

Now write $J = nqU_d$. We know $q$, we can obtain $n$ from the sample problem, so we can determine $U_d$:

$$U_d = \frac{J}{nq} = \frac{i}{nqA} = \frac{300 \text{ A}}{8.5 \times 10^{25} \text{ m}^{-3} \times 1.6 \times 10^{-19} \text{ C} \times 2.1 \times 10^{-5} \text{ m}^2}$$

$$= 1.0 \times 10^{-3} \text{ m s}^{-1}.$$

So it takes time $t = l/U_d = \frac{0.85 \text{ m}}{1.0 \times 10^{-3} \text{ m s}^{-1}} = 850 \text{ s}$ for electrons or $13.5 \text{ min}$ to travel length of the conductor.

Problem 2, 27-24

Start with $R = \rho l/A$, (wait, hold that thought)

a) Want $i$, $i = \Delta V/R = \frac{35.8 \text{ V}}{935 \Omega} = 0.04 \text{ A}$

b) $J = i/A = 0.04 \text{ A}/3.5 \times 10^{-4} \text{ m}^2 = 1.1 \times 10^2 \text{ A m}^{-2}$

c) $J = nqU_d$, $U_d = \frac{1.1 \times 10^2 \text{ A m}^{-2}}{5.33 \times 10^{22} \text{ m}^{-3} \times 1.6 \times 10^{-19} \text{ C} \times 4}$

$$= 1.3 \times 10^{-2} \text{ m s}^{-1}.$$
Problem 3 (cont.)

b) Now, if there is no taper, \( b = a = r \), \( R = \frac{q L}{\pi r^2} \)
which clearly is just \( \frac{q L}{A} \).

Problem 4, 28-14

The circuit has 3 sources of resistance:
internal resistance of battery, Cables, Starter.

A circuit diagram is shown below:

Now, \( V = E - i R_{\text{int}} \)
So \( R_{\text{int}} = \frac{(E - V)}{i} \)

So \( R_{\text{int}} = \frac{12V - 11.4V}{50A} = 0.012\Omega \)
battery is OK

Similarly \( R_c = \frac{\Delta V_c}{i} \)
So \( R_c = \frac{3.0V}{50A} = 0.06\Omega \)
Cable maximum is 0.04\( \Omega \), Cable is OK.

So, the potential difference across the starter is
\( 11.4V - 3.0V = 8.4V \). Then \( R_s = \frac{8.4V}{50A} = 0.17\Omega \)
Starter is OK.
Problem 5, 28-26

a) Can rewrite the circuit

Write \( R_{12} = R_1 + R_2 \)

\( R_{45} = R_4 + R_5 \)

And draw circuit over:

\[
\begin{array}{c}
\text{F} \\
R_1 \\
\text{R}_{12} \\
\text{R}_3 \\
\text{R}_4 \\
\text{R}_{45} \\
\text{R}_5 \\
\text{H} \\
\end{array}
\]

\[
\text{Then } R_{eq} = \left( \frac{1}{R_{12}} + \frac{1}{R_3} + \frac{1}{R_{45}} \right)^{-1}
\]

\( R_{12} = R_{45} = 10\,\Omega \), so \( R_{eq} = \left( 2 \times \frac{1}{10\,\Omega} + \frac{1}{5\,\Omega} \right)^{-1} = \left( \frac{2}{5\,\Omega} \right)^{-1} \)

or \( R_{eq} = 2.5\,\Omega \)

b) Again, re-write circuit:

Using same resistor numbering as in part a.

\[
\begin{array}{c}
\text{F} \\
R_1 \\
\text{R}_3 \\
\text{R}_4 \\
\text{R}_5 \\
\text{R}_{2345} \\
\text{G} \\
\end{array}
\]

First \( R_{45} = R_4 + R_5 \), then \( R_{345} = \left( \frac{1}{R_3} + \frac{1}{R_{45}} \right)^{-1} \)

Then we can redraw:

\( R_{2345} = R_2 + R_{345} \)

Finally, \( R_{eq} = \left( \frac{1}{R_1} + \frac{1}{R_{2345}} \right)^{-1} \)
Problem 5 (cont.)

Plug in numbers. \( R_{45} = 10 \Omega \), \( R_{345} = \left( \frac{1}{5\Omega} + \frac{1}{10\Omega} \right)^{-1} = 3.33 \Omega \)

\( R_{2345} = 8.33 \Omega \), \( R_{eq} = \left( \frac{1}{8.33\Omega} + \frac{1}{5\Omega} \right)^{-1} = 3.12 \Omega \)

Problem 6, 28-38

I have drawn the circuit without the switches for convenience. When switch is open we take relevant \( R \rightarrow \infty \).

\[\begin{align*}
&\text{a) with motor off, } V = 12V, I = 10A. \text{ If } R_s = \infty \\
&\text{then the current } i \text{ through the battery is just } i_L, \\
&\text{that through the lights. Then, there's a potential drop } \Delta V = -iR_{\text{int}} \text{ in the battery.}
\end{align*}\]

Thus, if \( E - V - iR_{\text{int}} = 0 \) \( E = V + iR_{\text{int}} \)

So \( E = 12V + 10A \times 0.05\Omega = 12.5V \)

\[\begin{align*}
&\text{b) We need to find resistance of lights, can get from provided information } R_L = \frac{12V}{10A} = 1.2\Omega.
\end{align*}\]
Problem 2 (cont.)

d) Now use $R = \rho l / A$ to find $\rho$. Then we will determine $E$ from $E = \rho J$.

$$\rho = \frac{RA}{l} = \frac{935 \Omega \times 3.5 \times 10^{-4} \text{m}^2}{0.158 \text{m}} = 2.1 \Omega \text{m}$$

Then $E = 2.1 \Omega \text{m} \times 1.1 \times 10^2 \text{Am}^{-2} = 228 \text{ Vm}^{-1}.$

I actually could have found this faster just using

$$E = \frac{\Delta V}{\ell} = \frac{35.8 \text{V}}{0.158 \text{m}} = 227 \text{ Vm}^{-1}$$

Consistent within rounding error

Problem 3, 27-29

The key insight in this problem is that the current through any slice of the resistor (I to its axis) is the same as the current flowing into or out of the resistor

$\rightarrow$ as $A$ increases, $J$ decreases.

First, write radius as a function of length,

$$r(l) = a + (b-a) \frac{l}{L} = a(1-\frac{l}{L}) + b\frac{l}{L}$$

I will solve the problem two different ways, using a “microscopic” approach and a “macroscopic” approach.
Problem 3 (cont.)

Microscopic approach: take a slice $dl$, calculate $E(l)$, then integrate to get $\Delta V$, $R = \Delta V/i$

So $J(l) = i/A(l) = i/\pi r^2(l)$, $E = J\rho = \frac{\rho i}{\pi (a + (b-a)\ell/L)^2}$

$\Delta V = -\int E\cdot d\ell = -\int E(l)dl$

So $\Delta V = -\frac{\rho i}{\pi a^2} \int_0^L \frac{1}{(1 + (b-a)\ell/L)^2} d\ell$

\[
= -\frac{\rho i}{\pi a^2} \left[ \frac{L}{1 + (b-a)\ell/L} \right] \bigg|_0^L \\
= -\frac{\rho i}{\pi a^2} \frac{L}{b-a} \left( \frac{a}{b} + 1 \right) = -\frac{\rho i}{\pi a^2} \frac{L}{b-a} (b-a)
\]

or $\Delta V = -\frac{\rho i L}{\pi ab}$ then $R = \Delta V/i = \frac{PL}{\pi ab}$

Macroscopic Approach: Take slice $dl$, calculate resistance then treat all slices as resistors in series. $\Sigma R \Rightarrow \int d\ell \rho / A$

So $R = \frac{\int d\ell \rho}{\pi r^2(l)}$ → Same integral as above but with $i$ removed $\Rightarrow$ get same answer as above for resistance.
Problem 6 (cont.)

Now, with the starter running, the potential drop across the lights is \( iR_L = 8A \times 1.2\Omega = 9.6\ V \) from the voltmeter.

Since \( E - V - iR_{int} = 0 \), the total current being drawn from the battery is \( i = \frac{(12.5\ V - 9.6\ V)}{0.05\ \Omega} = 58A \).

Since the lights draw 8A, the starter must draw 50A for charge (current) to be conserved.

Problem 7, 28-40

The part of the circuit that matters looks like:

There is a potential difference \( V \) across both of the resistors.

\[ i' = \frac{V}{R_v} \quad \text{so} \quad i' = i + iV = \frac{V}{R} + \frac{V}{R_v} \]

or \( i' = V \left( \frac{1}{R} + \frac{1}{R_v} \right) \)

the apparent Resistance \( R' \) is then, \( R' = \frac{V}{i'} = \left( \frac{1}{R} + \frac{1}{R_v} \right)^{-1} \)

or \( \frac{1}{R'} = \frac{1}{R} + \frac{1}{R_v} \Rightarrow \frac{1}{R} = \frac{1}{R'} - \frac{1}{R_v} \)
Problem 8, 28-55

Charge \( q \) on capacitor is zero at \( t=0 \)

So, there is a potential difference of \( V \) across \( R_2 \) & \( R_3 \). Since \( V_c=0 \)

a) So, initially we can treat the problem as if there is no capacitor. Then, the current \( i \) can be determined from \( E \) and the equivalent resistance \( R_{123} \).

\[
R_{123} = R_1 + R_2 + R_3 = R_1 + \left( \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1} = R_1 + \frac{R_2 R_3}{R_2 + R_3}
\]

but since \( R_2 = R_3 = 0.73 \text{ k}\Omega \), \( R_{123} = \frac{3}{2} \times 0.73 \text{ k}\Omega = 1.1 \text{ k}\Omega \)

then \( i = \frac{1.2 \text{ kV}}{1.1 \text{ k}\Omega} = 1.1 \times 10^{-3} \text{ A} \)

so \( V = E - IR_1 = 1.2 \text{ kV} - 1.1 \times 10^{-3} \text{ A} \times 0.73 \text{ k}\Omega = 400 \text{ V} \)

then the currents in \( R_2, R_3 \) are \( i_2 = i_3 = \frac{400 \text{ V}}{0.73 \text{ k}\Omega} \)

or \( i_2 = i_3 = 5.4 \times 10^{-4} \text{ A} \).

Now, at long times the capacitor has fully charged.

so \( V_c = V + i_3 = 0 \).
Problem 8, (cont.)

a (cont) if \( i_3 = 0 \), we can write the equivalent resistance as
\[
R_{eq} = R_{12} = R_1 + R_2
\]
\[
i = \frac{\varepsilon}{(R_1 + R_2)}
\]
\[
= \frac{1.2 \text{ KV}}{2 \times 0.73 \text{ m}\Omega} = 8.2 \times 10^{-4} \text{ A}
\]

then \( V = \varepsilon - i R_1 = 1.2 \text{ KV} - 8.2 \times 10^{-4} \text{ A} \times 0.73 \text{ m}\Omega = 600 \text{ V} \)

so \( i_2 = \frac{600 \text{ V}}{0.73 \text{ m}\Omega} = 8.2 \times 10^{-4} \text{ A} \) (I didn't need to do this since \( i_2 = i \))

Summary: @ \( t=0 \)
\[
i_1 = i = 1.1 \times 10^{-3} \text{ A}
\]
\[
i_2 = i_3 = 5.4 \times 10^{-4} \text{ A}
\]

@ \( t=\infty \)
\[
i_1 = i_2 = 8.2 \times 10^{-4} \text{ A}
\]
\[
i_3 = 0
\]

b) the potential \( V_2 \) across \( R_2 \) is given by \( V_2 = i_2 R_2 \)

Now, the current in the 3rd resistor is going to decrease exponentially with time. As we observed above, there is a corresponding increase in \( i_2 \) which we would expect to take a similar, exponential form. Though you are not required to do so, I will show you that this assumption is true.

Current/charge conservation requires \( i = i_1 = i_2 + i_3 \) ①
Because $R_2 \neq R_3/Capacitor$ see the same potential difference,

$$i_2R_2 = i_3R_3 + \frac{q}{C} \quad \text{But} \quad R_1 = R_2 = R_3 = R$$

So

$$i_2 = i_3 + \frac{q}{RC}$$

Plug in $0$:

$$-i_3 + i_1 = i_3 + \frac{q}{RC} \Rightarrow -i_1 + 2i_3 + \frac{q}{RC} = 0$$

We also have the requirement that $\sum_{\text{loop}} \Delta V = 0$

So

$$\epsilon - i_1R - i_3R - \frac{q}{C} = 0 \Rightarrow i_1 + i_3 + \frac{q}{RC} = \frac{\epsilon}{R} = 0$$

Now add (2) + (3):

$$\begin{align*}
    i_1 + i_3 + \frac{q}{RC} - \frac{\epsilon}{R} &= 0 \\
    -i_1 + 2i_3 + \frac{q}{RC} &= 0 \\
    3i_3 + 2\frac{q}{RC} - \frac{\epsilon}{R} &= 0
\end{align*}$$

This is the form I solved in class, $i_3 + \frac{q}{3RC} - \frac{\epsilon}{3R} = 0$

Simply from units, can identify $\gamma = 3\alpha RC$

Then, we saw that $i_3$ & $q$ will take the forms:

$$q = \frac{\epsilon}{3R} \gamma \left( 1 - e^{-t/\gamma} \right) = \frac{E_C}{2} \left( 1 - e^{-t/\gamma} \right)$$

$$i_3 = \frac{\epsilon}{3R} e^{-t/\gamma}$$

But $i_2 = i_3 + \frac{q}{CR} = \frac{\epsilon}{3R} e^{-t/\gamma} + \frac{E_C}{2RC} \left( 1 - e^{-t/\gamma} \right)$

$$= \frac{\epsilon}{2R} - \frac{1}{6} \frac{\epsilon}{R} e^{-t/\gamma} = \frac{\epsilon}{2R} \left( 1 - \frac{1}{3} e^{-t/\gamma} \right)$$

Check:

At $t=0$, $i_2 = \frac{\epsilon}{3R} = \frac{\epsilon}{3R} \times \frac{\gamma}{3} = \frac{\epsilon}{3R} = 5.4 \times 10^{-4} A$

At $t=\infty$, $i_2 = \frac{\epsilon}{2R} = 8.2 \times 10^{-4} A$
since \( V_2 = i_2 R \), \( V_2 = \frac{E}{2} \left( 1 - \frac{1}{3} e^{-t/\tau} \right) \)

at \( t = 0 \) \( V_2 = \frac{1}{3} E \), at \( t = \infty \) \( V_2 = \frac{1}{2} E \)

\( \tau = \frac{3}{2} RC \)
\( = 7.1 \text{ s} \)

Physical meaning of \( t \to \infty \) is that enough time-constants have elapsed such that \( V_2 \) is approximately at its asymptotic value.

i.e. \( \frac{V_{\text{asym}} - V_2(t)_{\infty}}{V_{\text{asym}}} = \frac{1}{3} e^{-t/\tau} \)

if \( t/\tau > 2 \), \( V_2 \) will be within 5% of its asymptotic value.
if \( t/\tau > 5 \), \( V_2 \) " " " "
0.2% of its asymptotic value.