Experiment 1-4:

Geometrical Optics I: Introduction

1. Introduction

This experiment is the first in a sequence of three experiments introducing the basic ideas of geometrical optics. These experiments are unrelated to the material you cover in class this semester. They are designed to present and apply the ideas of geometrical optics from an empirical point of view.

In this first experiment, we exploit a simple model describing light as a bundle of rays. We then explore the behavior of these rays as they reflect on smooth surfaces (mirror) and as they are transmitted, or “refract”, through transparent media. We experimentally verify Snell’s law of refraction. Finally, the phenomenon of total internal reflection is seen as a consequence of Snell’s law.

Why is optics important (besides the fact that the human eye works by the rules we describe here)? Optical instruments are used in many real-life situations in which an image of a system is needed. And such cases apply whether the system is large or small, and whether easily accessible or not. For example, whenever you want insight into how a biological system works, you will likely need to use imaging methods based on geometrical optics. Even if you are only interested in the final data from some fancy imaging system, it will be essential for the quality of your work that you know how to evaluate what you are seeing and what the limitations of the method are.

Remark:
It is strongly recommended that you look over the setup in the lab library before coming to the lab. This lab has many subparts and getting stuck at one point may make it difficult to finish the lab. Second, it is difficult to describe optical phenomena in the abstract, you simply must see them. (Excuse the pun!) So be sure to stop by the lab library and let the TA show you the apparatus!

* The material in these labs is discussed in the assigned lecture course textbook (Sears/Zemansky/Young, parts of chapters 36-38 inclusive.) Next semester, you will study topics in both geometrical and physical optics in lecture, which include interference and diffraction phenomena. Both geometrical and physical optics are understood as direct consequences of the wave nature of light. It is the “wavelength” of the light waves which is responsible for the different colors that we see. (See section 2.3.)
2. Theory

2.1 Geometrical Optics

Geometrical optics describes well light phenomena in which diffraction and interference effects are small. So as long as we do not consider cases where light passes through small pinholes or slits, or examine the edges of shadows, we are safe describing optical phenomena with geometrical optics. The simple assumption used for geometrical optics is that rays of light propagate along straight lines until they get reflected, refracted, or absorbed at a surface. A very simple, but important, application of geometrical optics is x-rays. The shadow image of the skeleton made in an x-ray may be understood by simply propagating straight lines from the source to the detector, except for those rays absorbed by some tissue (e.g. bone) in the path.

2.2 Light as Rays

The concept of light rays may be more plausible to you if you recall being in a dark, dusty room with light from the outside entering through a small pinhole. You can “see” a ray of light travelling in a straight line from the pinhole to wherever it hits the wall. (Your eye actually sees the light scattered by dust particles along the path of the ray.)

We describe light using the following simple model. A light source emits rays of light in all possible directions. Each ray propagates on a straight line until one of the following happens. It is

1. absorbed if it hits a non-transparent, non-reflecting material.
2. reflected if it strikes a reflective material.
3. refracted if it hits a transparent material.

For most materials in the real world, combinations of these effects occur.

In geometrical optics, we assume each ray of light travels along a straight line indefinitely unless it strikes a boundary, like a mirror or an interface between air and another material. A source of light, such as a flashlight, produces many rays that have traveled nearly parallel to each other on their way to the illuminated spot on the boundary. If a ray is not absorbed when it strikes the boundary, it splits into parts as shown in the figure above. Both the reflected and refracted rays again travel in straight lines until they en-
counter another boundary. The angles $i$, $r$, and $r'$ are measured with respect to the normal (i.e. perpendicular) to the boundary surface.

If the boundary is a metallic mirror, only the reflected ray is relevant. In sections 2.4 and 2.6, we describe the quantitative relationships between the incident ray and the reflected and refracted rays. In particular, we write the relations between the angles $i$ and $r$, and between the angles $i$ and $r'$ shown in the figure.

2.3 Absorption/Color (slight aside)

The colors we perceive in light are directly correlated with the wavelength of the light ray. White light is the response your brain perceives if light of all colors strikes your eye’s receptors.

Materials that absorb light can either decrease the reflected intensity for all colors equally (which means that the light simply appears less bright than before) or they can selectively decrease the intensity of some colors. An example of the second case is white light from the sun falling onto the grass where the grass absorbs all colors except green (which it reflects). This is why grass appears to you as green. If an object absorbs all the light, it appears to be black.

All materials absorb to some extent, even when the light appears to pass through or reflects. (The best commercial mirrors reflect about 99.99% of the incoming light.)

2.4 Reflection

The reflected ray from a surface emerges at an angle equal to the angle of incidence. Quantitatively, this law of reflection is expressed in terms of the angles shown in the figure as

$$r = i.$$

There are two general types of reflection - diffuse and specular. Diffuse reflection is exemplified by the reflection from the surface of this page. On a microscopic scale, the surface of the paper is quite rough; consequently, parallel rays striking even nearby parts of the paper’s surface are characterized by different angles of incidence. Each of the initially parallel rays is reflected in a different direction. Therefore, when we shine a flashlight on a piece of paper, we see a bright spot on the paper* whenever we stand to look at the paper. In this case, it is not feasible to determine the relationship between $i$ and $r$ for any particular ray.

If parallel rays reflect from a surface which is very flat and smooth (such as that found on a mirror or on window glass), there is a unique angle of incidence and hence a unique angle of reflection. This type of reflection is referred to as specular or regular. A flashlight

* Only a few of the light rays are reflected into our eyes!
beam reflected specularly will only be observed if it is viewed along the direction of reflection.

We restrict our attention here to specular reflection, since only in that case the relationship between the incident and reflected rays can be understood well enough to be used in optical instruments.

You may object that you have seen fresh snow glitter in the sunlight; which type of reflection is this? The explanation is that some of the small flat surfaces of the snowflake are smooth and act like mirrors that reflect several rays in the same direction, which just happens to be where your eye is! But the totality of the surface of white snow is irregular and reflection from the snow more often looks much like the diffuse reflection from this page.

*Remark for Experts:*

There is of course no perfectly smooth surface in nature. Every surface is rough on the atomic level. To get specular reflection, it is sufficient that surface irregularities are small compared to the wavelength of light. So when you polish anything, from optical instruments to your furniture, the desired successful appearance is accomplished when you have made all surface irregularities small compared to the wavelength of light (which is about $5 \times 10^{-5}$ cm). Since atoms are much, much smaller than that, there is no contradiction.

### 2.5 Real and Virtual Images

When you look in a mirror, you see your own image. It looks as if a copy of you is standing behind the mirror. But if you put a photoplate or a screen behind the image at the position where your copy seems to stand, you would obviously not obtain a photograph of your copy. Your copy in the mirror, and any image you cannot capture on a photo plate at its apparent location is called a virtual image. If you can record the image on a screen at its apparent location, it is called a real image.

Mirrors produce virtual images. In the next two labs, we will deal with many different examples of real and virtual images.

### 2.6 Refraction and Snell’s Law

Light waves (rays) propagate through the vacuum (or air) with a fixed velocity $c$ equal to about 300,000 km/s or $3 \cdot 10^8$ m/s. One of the consequences of our understanding of electromagnetism, detailed in Einstein’s theory of relativity, is that nothing can travel faster than this speed.

But as the waves travel through transparent material, interactions of the light with the atoms result in a velocity $v$ which is smaller than $c$. 
The ratio of these speeds is called the index of refraction $n$, and is a specific constant associated with the medium,

$$n = \frac{c}{v}.$$ 

For optical wavelengths traversing most transparent media, the index varies between about 1 and 2.5, depending on the material. Typical glass, for example, has an index of refraction equal to about 1.5.

When a ray of light in air encounters a medium, its change in wave velocity requires the wavefront of the wave to change – this means that the direction of the ray must change. The new angle relative to the normal, shown in the figure, is the angle of refraction ($r'$) given by Snell’s Law

$$\sin r' = \frac{\sin i}{n}.$$ 

Since $n$ is always larger than unity, the refracted ray is closer to the normal than the incident ray. For example, if an incoming ray at an angle $i = 15$ degrees encounters glass (with an index of refraction of $n = 1.5$), then the refracted ray will traverse the glass from the interface at an angle of $r' = 9.9$ degrees relative to the normal.

### 2.7 Total Internal Reflection and Critical Angle

Snell’s law describes the paths of light rays whether the ray starts from the air and enters the glass or the reverse. In other words, if we reverse the directions of the arrows, our picture in the figure of section 2.2 still describes the path of the light ray (with the labeling of $i$ and $r'$ reversed, and the reflected ray going down into the glass). This means that, when light passes from glass to air, the exiting ray has a larger angle to the normal than the incident ray.

In other words, when passing from glass to air, Snell’s Law requires $n \sin i = \sin r'$. So at some incident angle the refracted ray travels along the surface; for angles beyond this, no refracted ray emerges. In other words, the ray is totally reflected back into the glass. This angle is called the critical angle, $\theta_c$, and is given from Snell’s Law by

$$\sin \theta_c = \frac{1}{n}.$$ 

An easy way to determine the index of refraction of a medium is by measuring the critical angle and using this equation. As you can see, total internal reflection can only happen when a light beam passes from an optically denser (higher index of refraction) to an optically thinner medium (lower index of refraction).

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* The directional change follows from a simple physical argument. This will be discussed next semester.
2.8 Method of Parallax

Recall the discussion of parallax back in section 2.2 of laboratory 2. In a nutshell, parallax occurs when the relative locations of two objects move as you change your own position to view them. In these optics labs, we use parallax to find the positions of virtual images. The idea is to determine the virtual image of a pin and put another pin at that position. Only if you exactly place it at the position of the virtual image will you get no parallax. The figure below illustrates how this is done.

The observer at position 1 sees pin 2 to the left of pin 1. As the observer moves to position two, however, pin 2 appears to move over to the right of pin 1. These apparent motions, parallax, are only removed if pins 1 and 2 are at the same location.

3. Experiments

3.1 Reflection

In the first part of the lab, we verify the law of reflection with a mirror.

We accomplish this by specifying a ray using two pins placed in front of a mirror. The two pins form a line; when your head is placed behind the pins so that you see them as a single pin, the direction of the incoming ray is specified. If you now move your head to the other side of the normal, you will find these two pins to lie on top of each other only when your eye is at the location of the reflected ray. With two new pins along this line of sight, you should have 4 pins (two on each side of the mirror normal) that are exactly on top of each other when viewed along either the incident or reflected directions. Measuring the incoming and reflected angle allows you to check the law of reflection.
3.2 Image from a plane mirror

In the second part, we find the virtual image of a mirror using two different methods. Place a pin in front of the mirror and put a second pin behind the mirror at the position at which you see the virtual image. If the second pin is precisely at the location of the virtual image, no parallax will occur between the pin behind the mirror and the mirror image. The pin behind the mirror appears atop the pin in the mirror and remains there even if you move your head.

The second method for locating the virtual image requires that we mark (with pins) two different reflected rays of the first pin in front of the mirror. We then extend these rays behind the mirror where their intersection is the location of the virtual image.

3.3 Multiple Reflections

Two mirrors produce multiple reflections.

First, two mirrors are located next to each other with a 90-degree angle between them. An object placed between the two mirrors should produce three images: one image from each mirror, plus an additional image between them, which is an image of an image, produced by reflections from both mirrors. Whereas the first two images will have left and right interchanged, the last image will not.

Something very different occurs if you place the two mirrors parallel and facing each other. When you locate a pin between them, you see a series of pin images in the mirror due to multiple reflections in the mirrors.

3.4 Snell's Law

In this part, we verify Snell's law and use it to measure the index of refraction of water. The picture shows the setup of the experiment. With pins, we again specify the rays by making sure the three pins line up. After taking several measurements, plot \( \sin i \) vs. \( \sin r' \). The slope of the best-fit line will provide the index of refraction of water. You should understand why this is true and make sure you know how to plot the data to obtain the measurement.
3.5 Total Internal Reflection and Critical Angle

Finally, we obtain the index of refraction of plexiglass (a form of plastic called Lucite in Europe) by measuring the critical angle at a planar interface with air. A plexiglass block with a small black groove is used as shown in the figures to the left.

In the top figure, ray 1 is partially reflected back into the Lucite and partially refracted into the air.

In the middle figure, ray 2 is partially reflected back into the Lucite and -- in this special case -- is partially refracted into the air parallel to the surface.

In the bottom figure, ray 3 is incident at an angle, which is too large for it to be refracted; it is totally reflected.

When you view the top of the block from different angles, you see the black mark at different positions. If you now look along the Lucite surface of Side B, as shown in the middle figure, and mark the ray location along side B, the angle between this point and the groove is the critical angle. From this, one can calculate the index of refraction for the Lucite.
4. Specifics of the Experiments

Equipment List

4.1 Reflection

- Draw a straight line on a piece of drawing paper and place the mirror on it. Caution: The reflection takes place on the backside of the mirror! Therefore make sure that the back edge lines up with the line.
- Specify an incoming ray by placing two pins in front of the mirror.
- Find the reflected ray by looking into the mirror and moving your head until the two pins appear exactly on top of each other.
- Place two more pins between your eye and the mirror so they are lined up with the other two pins. This specifies the reflected ray.
- Remove the mirror and the pins from the paper and draw the incoming and reflected ray using the pinholes in the paper.
- Do the two rays intersect at the reflecting edge of the mirror?
- Draw the normal to the mirror through the intersection vertex of the two rays.
- Measure the incident and reflected angles relative to the normal.
- Estimate a reasonable uncertainty for these angles, considering the technique used to construct and measure them.
- Are the two angles equal within uncertainty?
- What were the main errors performing this experiment?
• 4.2 Image from a Plane Mirror

- Take a new sheet of paper and again draw a straight line near the middle of the paper and place the reflecting surface of the mirror along it.
- Place a pin in front of the mirror.
- Take another pin to be used behind the mirror. (You need to look over the top of the mirror to see it.) Find the location where the second pin has no parallax with the mirror image of the first pin. (The new pin should coincide with the image in the mirror, even if you move your head left and right. When this occurs, the second pin is at the position of the virtual image.)
- Is the position found by the absence of parallax more accurate when you hold your head near to the mirror or far away?
- Remove the second pin and make a mark at its position.
- Now use two pins to mark a reflected ray of the first pin by aligning two new pins with the mirror image of the first pin.
- Align two more pins for an additional reflected ray at a different angle.
- Remove everything from the paper and draw the reflected rays. Extend them behind the mirror until they cross.
- The crossing point is the predicted position of the virtual image. Does it coincide with the position found using the parallax free technique?
- Now draw the lines connecting the intercepts of the reflected rays with the mirror plane and the initial pin in front of the mirror. This line shows the actual path the light took as it was reflected in the mirror. Give a simple geometric argument why the distance between the object and the mirror should exactly equal the distance between the virtual image and the mirror.
- What are the major sources of error in this part of the lab?

4.3 Multiple Reflections

- Place two mirrors at a right angle to each other with their reflecting surfaces facing in.
- Write your name on a small piece of paper and hold it up to the mirrors.
- How many images do you see?
- Your name could appear written backward or forward. Note which images appear which way.
- Explain why your name appears written forward in the middle image!
- Now place the two mirrors parallel, facing each other, with an appropriate space between them.
- Place a pin between the mirrors.
- As you look into one of the mirrors over the top of the other, you should see a row of images extending back into the distance. Explain why you see several images of the pin!
- Why do the more distant images appear dimmer?
4.4 Snell's Law

- Fill the semi-circular box with water and place it on a sheet of paper.
- Trace the flat edge of the box on the paper, and mark the paper at the exact center of the flat surface.
- Place one pin at the center of the flat surface.
- Place an object pin further away (see figure in section 3.4) and look through the water from the curved side of the box. Place a locator pin so that it appears lined up with the other two pins.
- Repeat this procedure until you have a total of 5 pairs of rays on the same diagram. (Be sure to label which refracted ray goes with which image.)
- Remove everything and draw the rays using the pinholes.
- Draw the normal to the edge of the box through the center pin.
- Measure the angles, relative to the normal, of all the incoming and refracted rays.
- Make the plot of \( \sin i \) vs. \( \sin r' \).
- Explain why your data points should now appear on a straight line.
- Make a best-fit line. What are the slope and intercept (and associated uncertainties)? What should the intercept be?
- Determine the index of refraction of water from the slope.
- What is the value you find in books?
- Calculate the index of refraction explicitly from each pair of rays in your data. Calculate the mean and estimate the uncertainty of a single measurement of the index of refraction.
- Does the book value fall within one or two uncertainties of the mean?
- Note the main sources of error!

4.5 Total Internal Reflection and Critical Angle

- Take the Lucite (described in section 3.5) and look along the top surface as parallel to the surface as feasible.
- Measure the angle between the position of the groove and where you see it on the top surface relative to the normal.
- Give the critical angle.
- With this angle calculate the index of refraction of the Lucite.
- How well can you repeat this measurement?
- Give the major sources of error!
5. Applications

Swimming pool experience:
The next time you go swimming, do the following experiment (safely!):
From underwater, look at people or trees outside the water. You will see that you can see
them when you view with your eyes at small angles relative to the normal
to the water
surface. But as you look at larger angles, you find that the water surface suddenly appears
weird, somewhat like liquid mercury. This occurs because the water surface reflects like
a mirror so that you are not able to observe anything outside the pool. This is a conse-
quence of total internal reflection. (If you will not be swimming for a while, you can also
check this by looking at the water-air surface of a fish tank from below.)

Medical diagnoses:
A standard method of medical imaging utilizes ultrasonic sound echocardiography, ultra-
sonic sound waves produce images of inner body organs (along with many other applic a-
tions). A wave of ultrasonic sound sent into the body is refracted and reflected as it
passes through parts of the body of different density. The wave reflected from a specific
organ is observed by a detector and produces an image. The manner in which the ultra-
sonic sound waves get reflected and refracted is similar to the case for light that we have
considered.
An important difference between light and sound waves is that soundwaves propagate
faster in water than in air. (With light, it is the other way around!) This means that, for
sound, water has a lower index of refraction than air! Therefore, total internal reflection
occurs going from air to water. To enable the sound waves to get into the body requires
having no air between the source and the body, usually accomplished by putting a layer
of gel between the sound head and the body. This also means you cannot send ultrasonic
sound trough air-filled organs (like the lungs).
The picture below shows an echocardiogram of a human heart:

LA = left atrium, LV = left ventricle, RA = right atrium, RV = right ventricle.
Picture from: Marvin Berger: Doppler Echocardiography in Heart Disease
6. Lab Preparation Problems

Absorption:
1. How many times does a ray of light get reflected on a parallel pair of mirrors (which reflects 99% of the incoming light) such that the total intensity is down to 50%?

Reflection:
2. Explain in your own words the difference between specular and diffuse reflection using the following system:
   On a day with no wind, the surface of the sea can be very smooth and shiny. On the other hand, the tops of the waves appear white in a strong storm.

Index of Refraction and Snell's Law:
3. What is the index of refraction if a medium can slow down light to $v = \frac{2}{3} c$?

4. If you measure an incoming ray from the air to be at an angle of 15 degrees to the normal, what angle will it have in the medium with an index of refraction of $n = 1.2$?

5. If you measure $i = 25$ degrees (air) and $r' = 20$ degrees, what is the index of refraction of the medium in which you measured $r'$?

6. Given the following pairs of values for $i$ and $r'$, draw a graph of $\sin r'$ vs. $\sin i$! Determine the value of $n$ using only the slope of the graph and not the individual values from the table!

<table>
<thead>
<tr>
<th>$i$ in degrees</th>
<th>0</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r'$ in degrees</td>
<td>0</td>
<td>11.5</td>
<td>22.6</td>
<td>33.0</td>
<td>41.8</td>
</tr>
</tbody>
</table>

7. Given the following measured values for the index of refraction, what is the mean and uncertainty (using the 2/3 rule)?

<table>
<thead>
<tr>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.24</td>
</tr>
<tr>
<td>1.21</td>
</tr>
<tr>
<td>1.29</td>
</tr>
<tr>
<td>1.23</td>
</tr>
<tr>
<td>1.27</td>
</tr>
<tr>
<td>1.26</td>
</tr>
</tbody>
</table>

Total Internal Reflection and Critical Angle:
8. Given an index of refraction of diamond $n = 2.42$, what is its critical angle?

9. You want to measure the index of refraction of water. For that you take a glass of water or go to your fish tank and look upwards at the air-water boundary through the side of your glass. When look at the boundary with a small enough angle, the surface will be (almost) 100% reflective and you cannot see out through the boundary. With this setup, you measure the critical angle to be 60 degrees. What does this data give for the index of refraction of water? Is it the same as the value you find in books?