Experiments
in
Physics

Physics 1291
General Physics I Lab

Columbia University

Department of Physics

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General Physics I Lab Manual

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<u>Introduction 1-0:</u>

<u>General Instructions</u>

1. Purpose of the Laboratory

The laboratory experiments described in this manual are an important part of your physics course. Most of the experiments are designed to illustrate important concepts described in the lectures. Whenever possible, the material will have been discussed in lecture before you come to the laboratory. But some of the material, like the first experiment on measurement and errors, is not discussed at length in the lecture.

Other exceptions will be the three experiments on geometrical optics (Experiments 4, 5, and 6), for which instruction will only be given in the laboratory (while you study mechanics in lectures). This is not because the material is unimportant but because geometrical optics, an area of physics permitting design of optical instruments, can be considered primarily from an experimental and practical point of view. The fundamental nature of light, with its dual wave and particle properties (physical optics), will be treated in lecture during the second semester.

The sections headed <u>Applications</u> and <u>Lab Preparation Examples</u>, which are included in some of the manual sections, are *not* required reading unless your laboratory instructor specifically assigns some part. The Applications are intended to be motivational and so should indicate the importance of the laboratory material in medical and other applications. The Lab Preparation Examples are designed to help you prepare for the lab; you will not be required to answer all these questions (though you should be able to answer any of them by the end of the lab). The individual laboratory instructors may require you to prepare answers to a subset of these problems.

2. Preparation for the Laboratory

In order to keep the total time spent on laboratory work within reasonable bounds, the write-up for each experiment will be completed at the end of the lab and handed in *before the end of each laboratory period*. Therefore, it is <u>imperative</u> that you spend sufficient time preparing for the experiment *before* coming to laboratory. You should take advantage of the opportunity that the experiments (except 3 and 8) are set up in the <u>Lab Library (Room 506)</u> and that TAs there are willing to discuss the procedure with you. A prior visit to the Lab Library is especially recommended for the optics experiments.

At each laboratory session, the instructor will take a few minutes at the beginning to go over the experiment to describe the equipment to be used and to outline the important issues. This does not substitute for careful preparation beforehand! You are expected to have studied the manual and appropriate references at home so that you are prepared when you arrive to perform the experiment. The instructor will be available primarily to answer questions, aid you in the use of the equipment, discuss the physics behind the experiment, and guide you in completing your analysis and write-up. Your instructor will describe his/her policy regarding expectations during the first lab meeting.

Some experiments and write-ups may be completed in less than the three-hour laboratory period, but under no circumstances will you be permitted to stay in the lab after the end of the period or to take your report home to complete it. If it appears that you will be unable to complete all parts of the experiment, the instructor will arrange with you to limit the experimental work so that you have enough time to write the report during the lab period.

Note: Laboratory equipment must be handled with care and each laboratory bench must be returned to a neat and orderly state before you leave the laboratory. In particular, you must turn off all sources of electricity, water, and gas.

3. Bring to Each Laboratory Session

- A pocket calculator (with basic arithmetic and trigonometric operations).
- A pad of 8.5 x 11 inch graph paper and a sharp pencil. (You will write your reports on this paper, including your graphs. Covers and staplers will be provided in the laboratory.)
- A ruler (at least 10 cm long).

4. Graph Plotting

Frequently, a graph is the clearest way to represent the relationship between the quantities of interest. There are a number of conventions, which we include below.

- A graph indicates a relation between two quantities, x and y, when other variables or parameters have fixed values. Before plotting points on a graph, it may be useful to arrange the corresponding values of x and y in a table.
- Choose a convenient scale for each axis so that the plotted points will occupy a substantial part of the graph paper, but do not choose a scale which is difficult to plot and read, such as 3 or 3/4 units to a square.
- Label each axis to identify the variable being plotted and the units being used. Mark prominent divisions on each axis with appropriate numbers.
- Identify plotted *points* with appropriate symbols, such as crosses, and when necessary draw vertical or horizontal *bars* through the points to indicate the range of uncertainty involved in these points.
- Often there will be a theory concerning the relationship of the two plotted variables. A linear relationship can be demonstrated if the data points fall along a single straight line. There are mathematical techniques for determining which straight line best fits the data, but for the purposes of this lab it will be sufficient if you simply make a rough estimate visually. *The straight line should be drawn as near the mean of the all various points as is optimal*. That is, the line need not precisely pass through the first and last points. Instead, each point should be considered as accurate as any other point (unless there are experimental reasons why some points are less accurate than others). The line should be drawn with about as many points above it as below it, and with the 'aboves' and 'belows' distributed at random along the line. (For example, not all points should be above the line at one end and below at the other end).

5. Error Analysis

All measurements, however carefully made, give a range of possible values referred to as an uncertainty or error. Since all of science depends on measurements, it is important to understand uncertainties and where they come from. Error analysis is the set of techniques for dealing with them.

In science, the word "error" does not take the usual meaning of "mistake". Instead, we will use it interchangeably with "uncertainty" when talking about the result of a measurement. There are many aspects to error analysis and it will feature in some form in every lab throughout this course.

5.1 Inevitability of Experimental Error

In the first experiment of the semester, you will measure the length of a pendulum. Without a ruler, you might compare it to your own height and (after converting to meters) make an estimate of 1.5m. Of course, this is only approximate. To quantify this, you might say that you are sure it is not less than 1.3m and not more than 1.7m. With a ruler, you measure 1.62m. This is a much better estimate, but there is still uncertainty. You couldn't possibly say that the pendulum isn't 1.62001m long. If you became obsessed with finding the exact length of the pendulum you could buy a fancy device using a laser, but even this will have an error associated with the wavelength of light.

Also, at this point you would come up against another problem. You would find that the string is slightly stretched when the weight is on it and the length even depends on the temperature or moisture in the room. So which length do you use? This is a problem of definition. During lab you might find another example. You might ask whether to measure from the bottom, top or middle of the weight. Sometimes one of the choices is preferable for some reason (in this case the middle because it is the center of mass). However, in general it is more important to be clear about what you mean by “the length of the pendulum” and consistent when taking more than one measurement. Note that the different lengths that you measure from the top, bottom or middle of the weight do not contribute to the error. *Error* refers to the range of values given by measurements of exactly the same quantity.

5.2 Importance of Errors

In daily life, we usually deal with errors intuitively. If someone says “I'll meet you at 9:00”, there is an understanding of what range of times is OK. However, if you want to know how long it takes to get to JFK airport by train you might need to think about the range of possible values. You might say “It'll probably take an hour and a half, but I'll allow two hours”. Usually it will take within about 10 minutes of this most probable time. Sometimes it will take a little less than 1hr20, sometimes a little more than 1hr40, but by allowing the most probable time plus three times this uncertainty of 10 minutes you are almost certain to make it. In more technical applications, for example air traffic control, more careful consideration of such uncertainties is essential.

In science, almost every time that a new theory overthrows an old one, a discussion or debate about relevant errors takes place. In this course, we will definitely not be able to overthrow established theories. Instead, we will verify them with the best accuracy allowed by our equipment. The first experiment involves measuring the gravitational acceleration g. While this fundamental parameter has clearly been measured with much greater accuracy elsewhere, the goal is to make the most accurate possible verification using very simple apparatus which can be a genuinely interesting exercise.

There are several techniques that we will use to deal with errors throughout the course. All of them are well explained, with more formal justifications, in "*An Introduction to Error Analysis*" by John Taylor, available in the Physics Library (currently housed within the Science Library in the Mathematics building).

6. Questions or Complaints

If you have a difficulty, you should attempt to work it through with your laboratory instructor. If you cannot resolve it, you may discuss such issues with:

- one of the laboratory Preceptors in Pupin Room 729;
- the Undergraduate Assistant in the Departmental Office – Pupin Room 704;
- the instructor in the lecture course, or the Director of Undergraduate Studies;
- your undergraduate advisor.

As a general rule, it is a good idea to work downward through this list, though some issues may be more appropriate for one person than another.

Experiment 1-1

Uncertainty and Error

1. Introduction

There is no such thing as a perfect measurement. All measurements have errors and uncertainties, no matter how hard we might try to minimize them. Understanding possible errors is an important issue in any experimental science. The conclusions we draw from the data, and especially the strength of those conclusions, will depend on how well we control the uncertainties.

Let's look at an *example*: You measure two values 2.5 and 1.5. From theory, the expected value is 2.3, so the value 2.5 almost agrees, whereas 1.5 is far off. But if you take into account the uncertainties (i.e. the interval in which your result is expected to lie), neither may be far off. For experimental uncertainties of 0.1 and 1.0, respectively, your two measured values may be expressed 2.5 ± 0.1 and 1.5 ± 1.0. The expected value falls within the range of the second measurement but not of the first!

This first lab deals exclusively with this important subject. The techniques studied here will be essential for the rest of this two-semester lab course. The issues are important in order to arrive at good judgments in any field (like medicine) in which it is necessary to understand not just numerical results, but the uncertainties associated with those results.

2. Theory

2.1 Types of Uncertainties

Uncertainty in a measurement can arise from three possible origins: the measuring device, the procedure of how you measure, and the observed quantity itself. Usually the largest of these will determine the uncertainty in your data.

Uncertainties can be divided into two different types: systematic uncertainties and random uncertainties.

2.1.1 Systematic Uncertainties

Systematic uncertainties or systematic errors always bias results in one specific direction. They will cause your measurement to consistently be higher or lower than the accepted value.

An *example* of a systematic error follows. Assume you want to measure the length of a table in cm using a meter stick. However, the stick is made of metal that has contracted due to the temperature in the room, so that it is less than one meter long. Therefore, all the intervals on the stick are smaller than they should be. Your numerical value for the length of the table will then always be larger than its actual length no matter how often or how carefully you measure. Another example might be measuring temperature using a mercury thermometer in which a bubble is present in the mercury column.

Systematic errors are usually due to imperfections in the equipment, improper or biased observation, or the presence of additional physical effects not taken into account. (An example might be an experiment on forces and acceleration in which there is friction in the setup and it is not taken into account!)

In performing experiments, try to estimate the effects of as many systematic errors as you can, and then remove or correct for the most important. By being aware of the sources of systematic error beforehand, it is often possible to perform experiments with sufficient care to compensate for weaknesses in the equipment.

2.1.2 Random Uncertainties

In contrast to systematic uncertainties, random uncertainties are an unavoidable result of measurement, no matter how well designed and calibrated the tools you are using. Whenever more than one measurement is taken, the values obtained will not be equal but will exhibit a spread around a mean value, which is considered the most reliable measurement. That spread is known as the random uncertainty. Random uncertainties are also unbiased - meaning it is equally likely that an individual measurement is too high or too low.

From your everyday experience you might be thinking, "Stop! Whenever I measure the length of a table with a meter stick I get exactly the same value no matter how often I measure it!" This may happen if your meter stick is insensitive to random measurements, because you use a coarse scale (like mm) and you always read the length to the nearest mm. But if you would use a meter stick with a finer scale, or if you interpolate to fractions of a millimeter, you would definitely see the spread. As a general rule, if you do not get a spread in values, you can improve your measurements by using a finer scale or by interpolating between the finest scale marks on the ruler.

How can one reduce the effect of random uncertainties? Consider the following *example.* Ten people measure the time of a sprinter using stopwatches. It is very unlikely that each of the ten stopwatches will show exactly the same result. Even if all of the people started their watches at exactly the same time (unlikely) some of the people will have stopped the watch early, and others may have done so late. You will observe a spread in the results. If you *average* the times obtained by all ten stop watches, the *mean* value will be a better estimate of the true value than any individual measurement, since the uncertainty we are describing is random, the effects of the people who stop early will compensate for those who stop late. In general, making multiple measurements and averaging can reduce the effect of random uncertainty.

Remark: We usually specify any measurement by including an estimate of the random uncertainty. (Since the random uncertainty is unbiased we note it with a ± sign). So if we measure a time of 7.6 seconds, but we expect a spread of about 0.2 seconds, we write as a result:

$$t = (7.6 \pm 0.2)\ \text{s}$$

indicating that the uncertainty of this measurement is 0.2 s or about 3%.

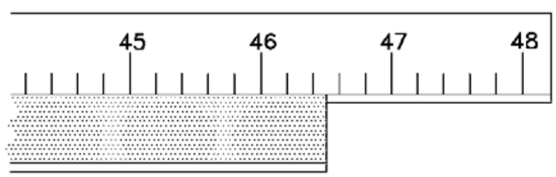
From here on, we use the term "uncertainty" to refer to random uncertainty, whereas systematic uncertainty will be specified as "error."

2.2 Numerical Estimates of Uncertainties

For this laboratory, we will estimate uncertainties with three approximation techniques, which we describe below. You should note which technique you are using in a particular experiment.

2.2.1 Upper Bound

Most of our measuring devices in this lab have scales that are coarser than the ability of our eyes to measure.



For example in the figure above, where we are measuring the length of an object against a meter stick marked in cm, we can definitely say that our result is somewhere between 46.4 cm and 46.6 cm. We assume as an *upper* bound of our uncertainty, an amount equal to *half* this width (in this case 0.1cm). The final result can be written:

$$l = (46.5 \pm 0.1)\ \text{cm}.$$

2.2.2 Estimation from the Spread (2/3 method)

For data in which there is random uncertainty, we usually observe individual measurements to cluster around the mean and drop in frequency as the values get further from the mean (in both directions).[1] Find the interval around the mean that contains about 2/3 of the measured points: *half* the size of this interval is a good estimate of the uncertainty in each measurement.

The reasons for choosing a range that includes 2/3 of the values come from the underlying statistics of the normal (or Gaussian) distribution. This choice allows us to accurately add and multiply values with errors and has the advantage that the range is not affected much by outliers and occasional mistakes. A range that always includes all of the values is generally less meaningful.

Example:

You measure the following values of a specific quantity:

9.7, 9.8, 10, 10.1, 10.1, 10.3

The mean of these six values is 10.0. The interval from 9.75 to 10.2 includes 4 of the 6 values; we therefore estimate the uncertainty to be 0.225. The result is that the best estimate of the quantity is 10.0 and the uncertainty of a single measurement is 0.2.[2]

2.2.3 Square-Root Estimation in Counting

For inherently random phenomena that involve counting individual events or occurrences, we measure only a single number N. This kind of measurement is relevant to counting the number of radioactive decays in a specific time interval from a sample of material, for example. It is also relevant to counting the number of left-handed people in a random sample of the population. The (absolute) uncertainty of such a single measurement, N, is estimated as the square root of N. As an example, if we measure 50 radioactive decays in 1 second we should present the result as 50 ± 7 decays per second. (The quoted uncertainty indicates that a subsequent measurement performed identically could easily result in numbers differing by 7 from 50.)

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[1] There is a precise mathematical procedure to obtain uncertainties (standard deviations) from a number of measured values. Here we will apply a simple "rule of thumb" that avoids the more complicated mathematics of that technique. The uncertainty using the standard deviation for the group of values in our example below is 0.2.

[2] Note that about 5% of the measured values will lie *outside* ± twice the uncertainty.

2.3 Number of Significant Digits

The number of significant digits in a result refers to the number of digits that are relevant. The digits may occur after a string of zeroes. For example, the measurement of 2.3 mm has two significant digits. This does not change if you express the result in meters as 0.0023 m. The number 100.10, by contrast, has 5 significant digits.

When you record a result, you should use the calculated error to determine how many significant digits to keep. Let's illustrate the procedure with the following *example*. Assume you measure the diameter of a circle to be d = 1.6232 cm, with an uncertainty of 0.102 cm. You now round your uncertainty to one or two significant digits (up to you). So (using one significant digit) we initially quote $d = (1.6232 \pm 0.1)$ cm. Now we compare the mean value with the uncertainty, and keep only those digits that the uncertainty indicates are relevant. Finally, we quote the result as $d = (1.6 \pm 0.1)$ cm for our measurement.

Suppose further that we wish to use this measurement to calculate the circumference c of the circle with the relation $c = \pi \cdot d$. If we use a standard calculator, we might get a 10 digit display indicating:

$$c = 5.099433195 \pm 0.3204424507 \text{ cm.}$$

This is not a reasonable way to write the result! The uncertainty in the diameter had only one significant digit, so the uncertainty of the circumference calculated from the diameter cannot be substantially better. Therefore we should record the final result as:

$$c = 5.1 \pm 0.3 \text{ cm.}$$

(If you do intermediate calculations, it is a good idea to keep as many figures as your calculator can store. The above argument applies when you <u>record</u> your results!)

2.4 Relative and Absolute Uncertainty

There are two ways to record uncertainties: the absolute value of the uncertainty or the uncertainty relative to the mean value. So in the example above, you can write $c = (5.1 \pm 0.3)$ cm or equally well c = 5.1 cm (1.00 ± 0.06). You can see that if you multiply out the second form you will obtain the first, since $5.1 \times 0.06 = 0.3$. The second form may look a bit odd, but it tells you immediately that the uncertainty is 6% of the measured value. The number 0.3 cm is the absolute uncertainty and has the same units as the mean value (cm). The 0.06 (or 6%) is the relative uncertainty and has no units since it is the ratio of two lengths.

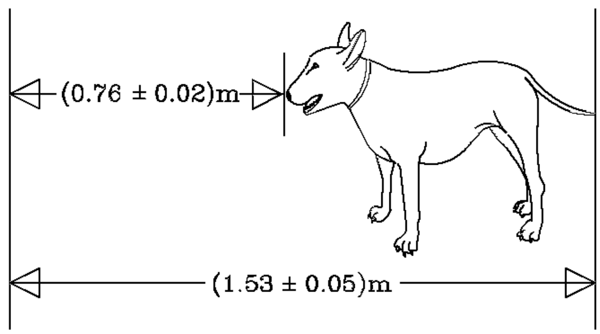
2.5 Propagation of Uncertainties

Often, we are not directly interested in a measured value, but we want to use it in a formula to calculate another quantity. In many cases, we measure many of the quantities in the formula and each has an associated uncertainty. We deal here with how to propagate uncertainties to obtain a well-defined uncertainty on a computed quantity.

2.5.1 Adding/Subtracting Quantities

When we add or subtract quantities, their uncertainties must always be added (never subtracted) to obtain the absolute uncertainty on the computed quantity.[3]

Take as an example measuring the length of a dog. We measure the distance between the left wall and the tail of the dog and subtract the distance from the wall to the dog's nose.



So the total length of the dog is:

$$\text{length} = (1.53 \pm 0.05)\text{ m} - (0.76 \pm 0.02)\text{ m}$$

$$= (1.53 - 0.76) \pm (0.05 + 0.02)\text{ m}$$

$$= (0.77 \pm 0.07)\text{ m.}$$

[3] The propagation of random uncertainties is actually slightly more complicated, but the procedure outlined here usually represents a good approximation, and it never underestimates the uncertainty. See "Remarks for Experts" at the end of section 2.5.2.

2.5.2 Multiplying/Dividing Quantities

When we multiply or divide quantities, we add (never subtract) the relative uncertainties to obtain the relative uncertainty of the computed quantity.

Take as an example the area of a rectangle, whose individual sides are measured to be:

$a = 25.0 \pm 0.5\ \text{cm} = 25.0\ \text{cm}\ (1.00 \pm 0.02)$

$b = 10.0 \pm 0.3\ \text{cm} = 10.0\ \text{cm}\ (1.00 \pm 0.03)$

The area is obtained as follows:

$$
\begin{aligned}
\textit{area} &= (25.0 \pm 0.5\ \text{cm})\cdot(10.0 \pm 0.3\ \text{cm}) \\
&= 25.0\ \text{cm}\ (1.00 \pm 0.02)\cdot 10.0\ \text{cm}\ (1.00 \pm 0.03) \\
&= (25.0\ \text{cm} \cdot 10.0\ \text{cm})(1.00 \pm (0.02 + 0.03)) \\
&= 250.0\ \text{cm}^2\ (1.00 \pm 0.05) \\
&= 250.0 \pm 12.5\ \text{cm}^2 \\
&= 250 \pm 10\ \text{cm}^2.
\end{aligned}
$$

Note that the final step has rounded both the result and the uncertainty to an appropriate number of significant digits, given the uncertainty on the lengths of the sides.

Remarks: Note that uncertainties on quantities used in a mathematical relationship always increase the uncertainty on the result. The quantity with the biggest uncertainty usually dominates the final result. Often one quantity will have a much bigger uncertainty than all the others. In such cases, we can simply use this main contribution.

Remarks for Experts: Our calculation of the uncertainty actually overestimates it. The correct method does not add the absolute/relative uncertainty, but rather involves evaluating the square root of the sum of the squares. For this lab, the simpler procedure described here will be adequate.

2.5.3 Powers and Roots

When raising a value to a certain power, its relative uncertainty is multiplied by the exponent. This applies to roots as well, since taking the root of a number is equivalent to raising that number to a fractional power.

Squaring a quantity involves multiplying its relative uncertainty by 2, while cubing a quantity causes its relative uncertainty to be multiplied by 3.

Taking the square root of a quantity (which is equivalent to raising the quantity to the ½ power) causes its relative uncertainty to be multiplied by 1/2.. For example, if you know the area of a square to be:

$$\text{area} = 100 \pm 8\ \text{m}^2 = 100\ \text{m}^2\,(1.00 \pm 0.08)$$

then it follows that the side of the square is:

$$\text{side} = 10\ \text{m}\ (1.00 \pm 0.04) = 10.0 \pm 0.4\ \text{m}.$$

You can convince yourself that this is true by checking it backwards using the rules described in section 2.5.2.

2.5.4 Multiplication by a Constant

Multiplying a value by a constant does not change its relative error.

2.5.5 Other Functions

If you need to calculate the error of a calculation that does not fit into one of these rules (such as trigonometric functions or logarithmic ones), here is a manual method that you can use.

Based upon the error of the quantity that you determined, you can find the maximum and minimum values of the quantity that you are calculating. The value that you found should be roughly midway between these two quantities. Then if you split the difference between the maximum and minimum you should obtain a reasonable estimate of the error.

Here is an example: Suppose you measure an angle to be (47.3 ± 0.5)° and you want to determine the error of sin(47.3 ± 0.5)°. You find that sin(47.3) = 0.735. Based upon your reported uncertainty, you know that your angle could be as large as 47.8° and as small as 46.8°, and therefore you should calculate sin(47.8) = 0.741 and sin(46.8) = 0.729. So your calculated value is 0.735 but it can be as low as 0.729 and as high as 0.741 and therefore, if you halve the difference between 0.729 and 0.741 you get a reasonable error estimate of 0.06. So you should report your value as 0.735 ± 0.006

3. Description and Procedure

You are to perform two experiments to practice the estimation of uncertainty and the propagation of errors. These involve measuring the period of a pendulum and measuring the reaction time of a human being.

3.1 Pendulum

In the first part, we measure the time it takes a pendulum to swing through a full cycle, the period of oscillation, and compare this to the theoretical prediction. Two methods of recording the time will be used to illustrate that different ways of making a measurement can result in very different uncertainties. In each case, many measurements will be taken to demonstrate the frequency of differing values. Since a number of measurements need to be made, you may wish to perform this experiment in teams of two: one person measures the time values, the other records the results.

In the first series, the stopwatch should be started when the pendulum is at one of its maximum positions and stopped as it returns to that position. A full period should be measured, not the time it takes for the pendulum to travel from one maximum point to the other. After 24 measurements, sort the measurements in bins of 50 ms. (Therefore all measurements with times between e.g. 1.25 s and 1.29 s (inclusive) are placed in the same bin.) Make a graph of the frequency of measurements in the bins vs. the times of the bins. Calculate the average and estimate the uncertainty using the 2/3 rule.

In the second series, measure the time as the pendulum passes a fixed characteristic mark in the background. For this you might use a chair or the leg of a table, or the lowest point of the pendulum swing. In order to make sure you are again measuring a complete period, you should take care to measure the time between when the pendulum passes through the same point in the same direction. Perform the same experiment, making 24 measurements, and sort the measurements in bins of 50 ms. Again graph your result and get the average and the uncertainty using the 2/3 estimate.

See if there is a substantial difference between the uncertainties of these two measurements and also if the measured values (i.e. the averages) coincide within uncertainty. In addition, measure the length l of the pendulum from the pivot point to the center of the mass (note that you will need to guess somewhat on the location of the center of the mass). This allows you to determine the acceleration due to gravity at the earth's surface, g, using the formula[4]:

$$\frac{2\cdot\pi}{t} = \sqrt{\frac{g}{l}}$$

[4] This will be covered in the 1201 lecture course. See, for example, Chapter 15 in Fundamentals of Physics, 8th Ed., by Halliday, Resnick & Walker.

where *t* is the measured period. Make sure to propagate uncertainty from *l* and *t*. Finally, compare your value of *g* to the accepted value:

$$g = 9.8 \text{ m/s}^2.$$

- Does the accepted value of *g* fall within your uncertainty?

- What are the sources of error?

3.2 Reaction Time

Next measure your reaction time to a specific stimulus. Perform this experiment with a partner, but every student should obtain his/her own data.

One student holds a meter ruler (vertically) at the upper end and the other student places two fingers around (but not touching) the 50 cm mark of the ruler. The first student (quietly) releases the ruler and the second student tries to grab it as soon as possible after he/she sees it released. Measure the distance *s* the ruler falls. After performing this experiment 10 times (for each student), look at the spread in the data and calculate the resulting uncertainty (using the 2/3 estimate). If you have a few values that are far off from all the other values (e.g. because you were irritated or distracted) you may decide to ignore them when you calculate your reaction time, but note in your lab report which measured points were not used.

Use the relation[5]:

$$s = \frac{1}{2} g \cdot t^2$$

to calculate your average reaction time and the corresponding uncertainty. This experiment is an example of an indirect measurement. You measure a quantity (here the distance the ruler is falling) that you do not care about directly, but is necessary in order to calculate the quantity you want to know (the reaction time).

You should report your reaction time in seconds in your lab report.

- Would it be reasonable to give your measured values with a precision measured in millimeters? Why or why not?

- How well could you get the same initial conditions for each try?

- Note the major sources of error.

- Suggest improvements to the lab or shortly describe a better experiment.

[5] See, for example, Chapter 2 in the Halliday, Resnick & Walker text.

4. Sample Data Sheet

4.1 Pendulum

Length of the pendulum = ± cm

From Max to Max

Using Reference Point

5.2 Reaction Time

1		5		9	
2		6		10	
3		7			
4		8			

5. Applications (for those interested in everyday relevance!)

You read of a certain test intended to indicate a particular kind of cancer. The test gives you a positive result for (80 ± 10) % of all persons tested who really have this kind of cancer (true positives). But the test also gives you a positive result for (2 ± 1) % of all healthy persons (false positives). Now you read a publication where the author performed this test on 10,000 workers that deal with a certain chemical. The author got 400 positive samples from these workers and claims that this is strong evidence that this particular chemical enhances the development of this kind of cancer since it is known from literature that only (1 ± 0.5) % of the population are expected to have this kind of cancer. How reliable is the claim of the author?

Numerical Answer:

If one assumes that the 10,000 workers would mirror the average population, then there should be:

$$10000 \cdot (0.010 \pm 0.005) = 100 \pm 50$$

persons having this cancer. Of them, the test gives:

$$(0.8 \pm 0.1) \cdot (100 \pm 50) = 80 \pm 50$$

positive results (true positives).

There are then 9900 ± 50 persons expected not to have this kind of cancer. Of them:

$$(0.02 \pm 0.01) \cdot (9900 \pm 50) \approx 200 \pm 100$$

give positive results (false positives).

The total number of positives in the average population is therefore 280 ± 150.

So how do you judge the author's conclusion of "strong evidence"?

If you wanted to design a new test using the same procedure but to arrive at a stronger conclusion, and you could either increase the rate of true positives or decrease the rate of wrong negatives, which would you choose?

Reference:

Paul Cutler: Problem Solving in Clinical Medicine, Chapter 5, Problem 5 (modified).

6. Lab Preparation Examples

Below are some questions to help you prepare. You will not be expected to answer all these questions as part of your lab report.

Estimation of Uncertainty:

1. You have the following distribution of measured values:

0			
1	I		
2	III		
3	IIIII	I	
4	IIIII	IIII	
5	IIIII	IIIII	IIIII
6	IIIII	IIIII	I
7	IIIII	III	
8	IIII		
9	II		
10	I		
	5	10	15

Estimate the uncertainty using the 2/3 estimate.

2. Estimate the mean and uncertainty of the following group of values:

   1.6 s, 1.3 s, 1.7 s, 1.4 s

3. In a radioactive decay you get 16 counts. What is the absolute uncertainty of the number of counts? What is the relative uncertainty?

4. In a radioactive decay you get 1600 counts. What is the absolute uncertainty of the number of counts? What is the relative uncertainty?

5. How many counts should you get so that the relative uncertainty is 1% or less?

Significant digits:

6. How many significant digits has $l = 0.0254$ m?

7. Write $t = 1.25578 \pm 0.1247$ s with two significant digits (in the uncertainty).

Propagation of Uncertainty:

8. For a pendulum with $l = 1.0 \pm 0.1$ m, you measure a period of $t = 2.0 \pm 0.2$ s.

What is the value of the earth's acceleration g?

9. You measure the volume of a box by measuring the length of the single sides. For the lengths of the single sides you get:

$a = 10.0 \pm 0.1$ cm $b = 5.0 \pm 0.2$ cm $c = 7.5 \pm 0.3$ cm.

What is the volume of the box (including uncertainty and units) in cm^3?

What is the volume of the box in m^3?

10. You measure the following quantities:

$A = 1.0 \pm 0.2$ m $B = 2.0 \pm 0.2$ m $C = 2.5 \pm 0.5$ m/s

$D = 0.10 \pm 0.01$ s $E = 100 \pm 10$ m/s^2

Calculate the mean and uncertainty of:

a) $A+B =$

b) $A-B =$

c) $C \cdot D =$

d) $C/D =$

e) $C \cdot D + A =$

f) $\frac{1}{2} E \cdot D + C =$

g) $A \cdot B/(A-B) =$

Include units! For e)-g) perform it step by step.

Relative and Absolute Uncertainty:

11. What is the relative uncertainty for $v = 12.25 \pm 0.25$ m/s?

12. What is the absolute uncertainty if the mean value is 120 s and the relative uncertainty is 5%?

13. Given the following measurements, which one has the highest absolute uncertainty and which one has the highest relative uncertainty?

$l = 10.0 \pm 0.2$ m $l = 10.0$ m (1.00 ± 0.03) $l = 12.5 \pm 0.25$ m

$l = 7.24$ m (1.00 ± 0.04)

14. Given the following measurements which one has the highest absolute uncertainty and which one has the highest relative uncertainty?

$l = 10.0 \pm 0.2$ m $t = 7.5 \pm 0.2$ s $d = 5.6$ cm (1.00 ± 0.04)

$v = 6.4 \cdot 10^6$ m/s (1.00 ± 0.03)

Caution: Don't get tricked!

Explanations:

15. Explain, using your own words, why the uncertainty decreases as you average over several measurements.

16. Explain, using your own words, the difference between uncertainty and error as you perform several measurements and average.

17. You measure the speed of light and get as a result $c = (2.25 \pm 0.25) \cdot 10^8$ m/s. The value you find in books is $c = 299\,792\,458$ m/s. Using these values explain the difference between the uncertainty of your measurement and its error!

Experiment 1-2

Forces

1. Introduction

Forces are an essential element in the study of physics. The concept of force is familiar: effects of forces on the human body are part of everyday experience. But force has a precise meaning; forces can be measured, combined, and evaluated in exact ways. This laboratory is intended to demonstrate a few of the many examples of natural and familiar forces discussed in the lecture course.

Today's experiments illustrate the nature of several forces and how they are measured. First, we deal with the vector nature of forces and note how a situation of zero net force at a point requires that all forces in each direction must balance. The forces arise from the gravitational attraction to the earth (weight) of objects, which are transmitted to a point through string tension.[1]

Next, we work with the force arising by stretching or contracting a spring. This kind of force has more general applications than springs, in that the "elastic" nature of collisions (like a tennis ball bouncing on the floor) arises from spring-like forces. You will measure the quantitative relationship between the force applied and the subsequent displacement of a spring.[2]

Finally, we deal with an aspect of force that has not yet been discussed in lecture: the lever. This topic will be covered in the course lectures on rotations. For the purposes of this lab, you should see how stability against rotation adds to the concept of force the new concept of "lever arm".[3] Lever arm is an important aspect of how forces work in many physical situations, like the forces applied by muscles on your skeleton.

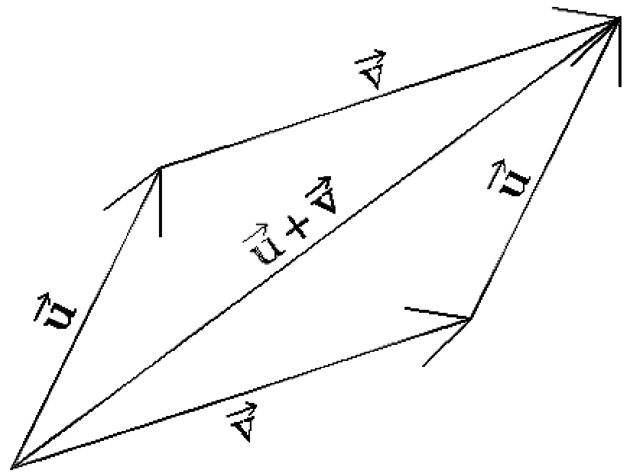
2. Subject Matter

2.1 Vector Forces

In the first part of the experiment, we measure three vector forces and show that when the system is stationary, the sum of all forces is zero. Vectors have length *and* direction, and therefore are usually represented by an arrow – its length corresponds to the magnitude of the vector, while its direction (from tail to head) corresponds to the direction of the vector. To add two vectors graphically, simply place the tail of one vector at the head of the other. Then start from the same origin and attach the two vectors in the reverse order, we must arrive at the same final point. From the parallelogram formed by this procedure, the

[1] See Chapters 5 and 12 of Fundamentals of Physics, 8th Ed., by Halliday, Resnick & Walker.
[2] See Section 7-7 of Halliday, Resnick & Walker.
[3] This will be covered in Chapter 11 of Halliday, Resnick & Walker.

arrow along the main diagonal represents the sum of the two vectors. (The other diagonal is the difference of the two vectors.)



Vectors can also be added algebraically. If a vector is described in a given basis, or specific coordinate axes, e.g. **u** = (1,2), you must add a second vector in the same basis. For example, if the second vector has components **v** = (3,-3), the components of the sum, or resultant, vector are **u+v** = (1+3,2+(-3)) = (4,-1).

Just as every two added vectors give a vector sum or resultant, the opposite is also true: every vector can be split into two "components" at right angles to each other. This is an important tool we use in physics repeatedly. You may choose, for the convenience of doing the problem, along which perpendicular directions to split the vector. There is usually a favorable choice of perpendicular basis vectors in which the problem can be described most easily.

2.1.2 Parallax

In the first part of this lab, we must draw images on paper of strings located above the paper. It will at first appear that there are many different possibilities of where to draw the images of the strings. If you draw lines on the sheet that seem directly below the strings when your head is in a given position, if you move your head a few centimeters left or right you will see that the lines are no longer covered by the strings. This effect, in which a closer object (here the string) seems to move relative to a distant background (here the sheet on which you draw the line) is called *parallax*. We will see effects of parallax several times during these labs, so it is important to learn how to handle it.

Where should the lines be drawn? A unique prescription for where to place the lines is provided by the instruction to *always look straight down on the string*. Now we must determine how we know when we are looking straight down on the string. Use a small mirror and place the mirror on the sheet directly under the string. Both the string and the image of the string in the mirror are visible. As your head moves left and right, the mirror image of the string moves relative to the string. When your head is located so that the string and its mirror image exactly overlap, *you are looking straight down*. Make two small marks beside the mirror where the image of the string enters and leaves the mirror. Then, remove the mirror and draw a line through the two points on the paper with a ruler. This line is the correct image of the string.

2.2 Springs

Ideal springs turn out to have a very simple relation between the force F applied to them and the distance s they are stretched. This *linear* relationship, called "Hooke's Law," is expressed by:

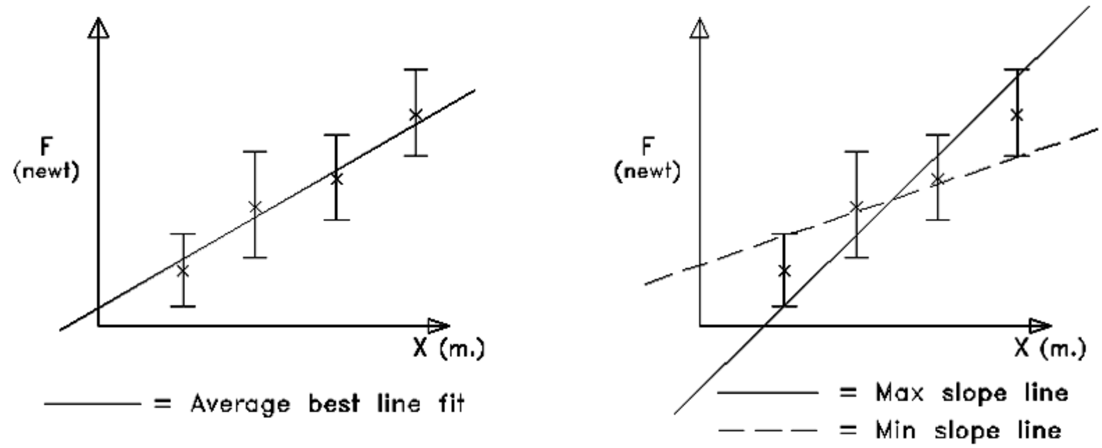
$$F = k \cdot s$$

where k is known as the "spring constant"[4]. It is the same for a specific spring, and its value depends only on the properties of that spring. In this lab, we will measure the forces necessary to stretch a string different distances, and from these we will determine the spring constant of the spring.

2.2.1 Best-Fit Line

In most research laboratories, plotting measurements on a graph is found to be the preferred method of reviewing the validity of data and quantitatively measuring the relationship between the variables. We often have some prior idea of the expected relationship between the variables. In these labs, this expected relationship is almost always arranged to be a straight line. But even if we know that ideal points fit on a precise straight line, real measured data points will not lie on a single line - because the measurements always have intrinsic uncertainty. Therefore when the points are plotted, we should include error bars to indicate the uncertainties in the measured points. Because real measurements do not all lie on a single straight line, there are a variety of possible lines you might put through the data points. Some examples are shown in the figure below.

[4] Note that this denotes the force applied *to* the spring, not by the spring.



Which line represents the best fit? There is actually an exact mathematical procedure to obtain the best-fit line, but since this is a tedious calculation (and usually done on a computer), we will introduce some useful "eyeball" methods that can be done easily, during the lab session without a computer. These techniques are sufficiently precise for our purposes, and with some experience they give results surprisingly close to the best computer fits.

How does one draw the best line? First, try to draw a line with as many points (with uncertainties included) lying above the line as below it. The gauge of how close the line is to a point is given by the uncertainty associated with that measured point. However, all the points at the left end should not lie on one side of the line with all the points at the right end lying on the other side. As a rule of thumb, roughly 2/3 of the points should have the line passing through the uncertainties. The uncertainty for the best-fit line is obtained by estimating how much one could increase and decrease the slope of the line before the fit is deemed very bad.

A second method to get the best-fit line is to draw minimum and maximum slope lines described above and then place the best-fit line exactly in the middle of these two lines. Minimum and maximum slope lines are drawn by estimating the largest and smallest possible slope that one can conceivably deduced from the graph. Half the difference between the minimum and maximum slopes is a good estimate of the slope uncertainty.

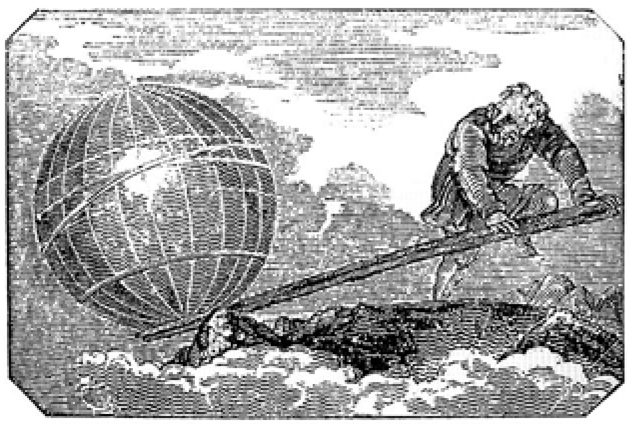
Remark: Often in our experiments the data points will not look as nice as in the above examples. One or several points may not be close to any best-fit line you try. Such anomalous points may occur, for example, because of a mistake in measuring. In such cases, it is acceptable to ignore these anomalies when estimating the best-fit line. (Of course you must note this fact down in your lab report) Dropping anomalous points must be done with extreme care and only rarely.[5] It is better to choose a line with as many points above the line as below. If you are not sure of your measurements, it is better to re-measure or to take more data points.

[5] More than once, data points that did not behave as theory predicted turned out to be new effects and led to Nobel prizes!

2.3 Lever Arm

A lever is a simple device that enables you to "trade force for distance" in a system subject to rotation about a pivot. You can choose to apply a smaller force to lift a body, so long as you move further away from the pivot or increase the "lever arm". Those who have enjoyed themselves in their younger years on a playground "see-saw" understand this principle well.

Among the first to explain the theory of levers, Archimedes is reported to have said, "Give me a place to stand and I will move the earth." This statement reflects the fact that with enough available distance, the force necessary to move the earth could be applied by a single person. The problem of course, is that there is nowhere to stand at that distance from the earth.



If two forces, F_1 and F_2, act on opposite sides of a pivot with lever arms (relative to the pivot) of s_1 and s_2, the lever will remain stationary if: [6]

$$F_1 \cdot s_1 = F_2 \cdot s_2.$$

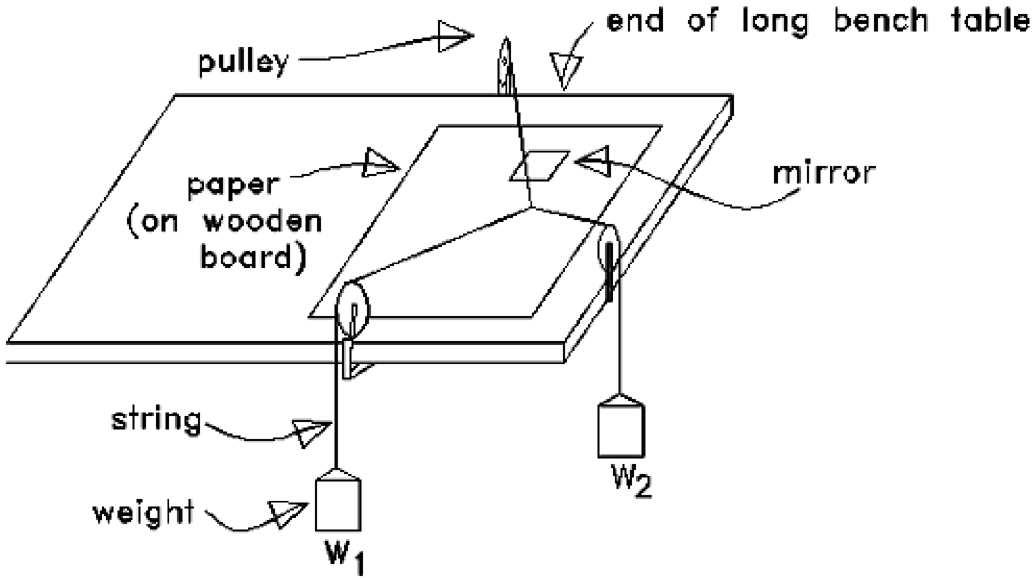
This implies that if F_1 and s_1 are fixed, then the product $F_2 \cdot s_2$ must be constant and equal to the product $F_1 \cdot s_1$, otherwise the lever will rotate.

3. Description and Procedure

3.1 Addition of Forces

In the first part of the experiment, we deal with three different forces. The force due to an unknown weight is balanced by two known forces. We will check the result by graphically adding the known forces. Using vector components, we check that the components of the three vectors in equilibrium cancel (within the uncertainty).

[6] The experts among you will realize that this merely requires that the two torques are equal. This will also be covered in lecture later in the semester.



The equipment includes a wooden board to which three pulleys are clamped. Over each pulley, we place a string with a weight. Choose two different known weights and attach them to two of the strings (you might begin with a *100 g* and *150 g* weight for example). The three strings are tied together over the board at a "node". Move the node, where the strings join, around gently and try to find the equilibrium position. (The position at which the node no longer moves, or the location to which the node returns if displaced a few cm, represent this equilibrium position.) Place a sheet of paper on the wooden board, under the strings. In order to have room to record the string locations, make sure that the node is approximately in the middle of your paper and the wooden board. (You may need to move the pulleys to accomplish this.)

You are going to draw vector representations of the forces due to each of the weights. Starting at the node, draw the three strings on your paper using the method of parallax-free reading. Then, draw two arrows along the lines corresponding to the strings holding the known masses, with lengths proportional to the masses on the strings. Choose a scale that is appropriate for your weights and the size of the paper. These arrows are vectors representing the forces due to the known weights.

Add these two vectors graphically using the parallelogram method described above to obtain their resultant force. The resultant of these two vectors should be opposite in direction but equal in length to the vector representing the unknown force (from the unknown mass).

- Are the two directions opposite? How far off are they?
- Measure the length of the resultant vector. From this length, calculate the magnitude of the unknown mass, and include uncertainty.

On the electronic scale, measure the unknown mass and compare to the mass determined by the parallelogram method.

- Are they the same? Should they be the same?

Now draw an arrow along the third (unknown) direction with length corresponding to the mass of the unknown weight you measured on the electronic scales.

- Specify the *x* and *y* components of all three vectors. Choose your *x*- and *y*-axis to make the work simplest!

- Determine if the sum of all the *x* components gives zero within uncertainty. Do the same for the *y* components. (Remember that components from two vectors pointing in opposite directions have opposite signs.)

- Note the main sources of error.

3.2 Spring

In the second part of the experiment we plot applied force vs. stretch of the spring, make a best-fit straight line and measure its slope. You should understand the physical interpretation of this slope.

<u>*General Comment*</u>: Every line is fully determined by its slope and intercept. Whenever you obtain a best-fit line, you should check if these two quantities are reasonable. Also, these two quantities often have a physical interpretation that you should understand.

Pick a spring and hang it from the pivot point. You will be using this spring for the entire experiment. Attach a *100 g* weight to the bottom of the spring. Adjust the measuring ruler so that a calibration mark lines up with the lower end of the spring.

Attach the force meter to the lower end of the spring and pull the spring so that it aligns with distances you want to use as measuring points. You can use the mirror from the first part of the experiment to get a parallax-free reading of the position of the spring. Your uncertainty in the position should then be as small as the resolution of the ruler.

Read the force meter to determine the force you are applying to the spring by pulling it, and estimate the uncertainty in the reading of the force meter.

- Record a total of 5 points with associated uncertainty.

- Draw a force vs. distance (stretch) graph. Don't forget the error bars!

- Draw a best-fit line through the data points. Draw the maximum and minimum slope on the graph. Do these in whichever order you find most convenient.

- Record the value of the slope and determine its error.   Interpret its physical meaning.

- What is the value of the intercept of your line (where the distance is zero)? How do you interpret this when it is non-zero?
- Give the value of the spring constant including uncertainty.
- Note the main sources of error.

3.3 Lever Arm

In the last part of the experiment, a pivoted lever is balanced to remain in horizontal equilibrium. The quantitative goal is to verify that force and distance are inversely proportional. (We check to see if *force* × *lever arm* = *constant* holds.) You will graph force vs. 1/distance for several distances from the pivot point. If a straight line can be reasonably fit to the data, then we can conclude that there is a linear relation between force and inverse distance. You may find that the curve has a non-zero *y*-intercept, due to the fact that we have not counted the weight of the force meter.

(NOTE: some of the lever arm setups have left and right inverted – examine your equipment to see whether you need to interchange left and right in the description below.)

Level the lever arm with the black paper clamp provided. Place the *500 g* weight on the single hook to the right of the pivot. Using the meter that measures Newtons, measure the force necessary to keep the lever in balance for each of the hooks on the right side of the pivot. Include the uncertainty associated with each recorded force!

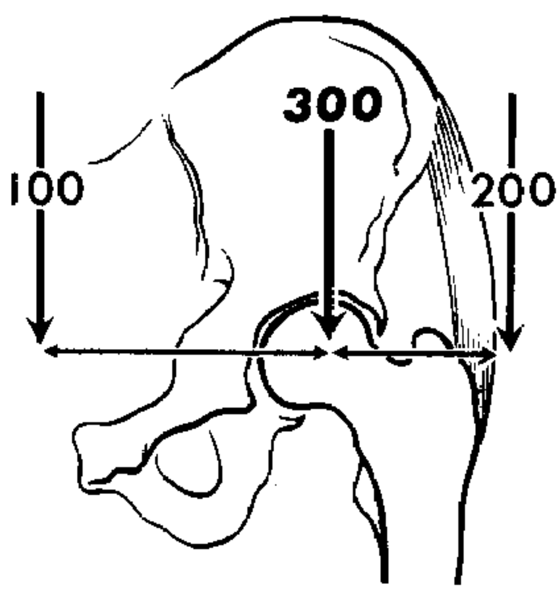
- Make a plot including each pair of data values: force vs. 1/lever arm. Include the error bars of the force measurement.
- Obtain the best-fit line in this graph. Comment on how well your best-fit line fits your measured data!
- Does your best-fit line go through the origin of the graph? Should it?
- According to your best-fit line, how much force would have been necessary if the lever arm were *10 meters* long (i.e. the balance force were applied at *s = 10 m*)?
- Note the main sources of error.

4. Application (to everyday experience)

How much force acts on your hip joint as you walk?

You may be tempted to respond that the force on the hip joint is simply the weight of the body above the hip. This is indeed the force acting down from above. However, when you walk (supported by one leg at a time) the force acting from below is actually complicated. The reason is that the supporting forces consist of (a) the reaction force of the leg bone (femur) directly supporting the hip, and (b) the forces transmitted by the muscles connecting the hip to the femur. The latter are necessary to keep the body in balance. In performing this function, the two kinds of forces actually operate in different directions and have very different lever arms. In the end, for a typical person of weight, W, with his/her weight on one leg at a time, the direct reaction force on the hip from the leg bone is about 2.4W acting upwards, but at an angle to the vertical. (So if you weigh 125 pounds, the total upward force on your hip from direct contact with the femur is over 300 pounds!) The force from the muscle attachments on the hip acts generally downwards (opposite direction) and is such as to balance the vertical forces and torques.[7]

So the balancing of forces (and torques) is an important element in dealing with the physiology of the human body.



[7] References: J. R. Cameron, et al, Physics of the Human Body (Medical Physics Publishing, Madison, 1992) p.36, and: R.M. Kenedi: Biomechanics and Related Bio-Engineering Topics.

5. Lab Preparation Examples

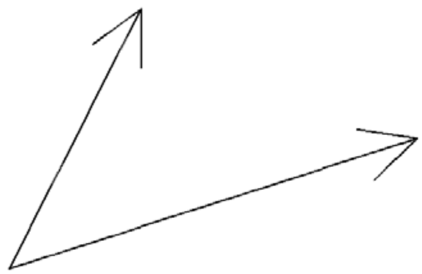
Vectors:

1. Add the following two vectors algebraically:

   u = (1,2), **v** = (2,-5).

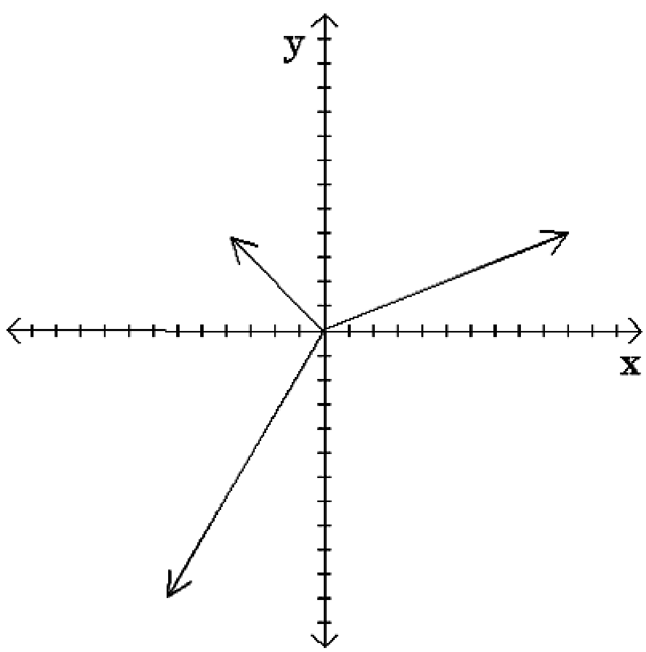
2. Draw the two vectors described above and add them graphically.

3. Graphically add the two vectors below:



4. Draw the two vectors **u** = (3,-5), **v** = (-1,7) and graphically check if their sum points in the same direction as the vector w = (-1,-1).

5. Split the vectors below into the two given components (x,y) and add them up:



6. Does the sum of the 5 forces $\mathbf{F_1},\ldots,\mathbf{F_5}$ below vanish?

   $\mathbf{F_1}$ = (1,2,3) N, $\mathbf{F_2}$ = (1,0,-5) N, $\mathbf{F_3}$ = (0,-3,7) N, $\mathbf{F_4}$ = (-3,1,0) N, $\mathbf{F_5}$ = (1,0,0) N

7. Show graphically that the sum of the three vectors **u** = (1,-2), **v** = (-3,4), **w** = (2,-2) vanishes.

8. Does the sum of the following three vectors vanish within uncertainty?

u = (10 ± 1, -5 ± 1), **v** = (-13 ± 1, 7 ± 1), **w** = (7 ± 1, -3 ± 1)

<u>Lever arm:</u>

9. You have a lever whose right hand lever arm is *1 m* and the left is *25 cm*. If you place a mass of *1 kg* on the left, how much mass do you need to place on the right so the lever stays balanced?

10. A force $F = 1.0 \pm 0.2$ N acts to the left of a pivot (length $L = 0.5 \pm 0.1$ m). The right side force acts at $R = 0.20 \pm 0.05$ m. What force must act on the right side if the lever is balanced?

11. You want to fix a flat tire on your car ($m = 1000$ kg). To lift your car you use a (massless) lever of total length *100 cm* pivoted about an unknown location along its length. How long should the distance from the car to the pivot be if you could only push down on the end of the lever with a force of *1000 N*?

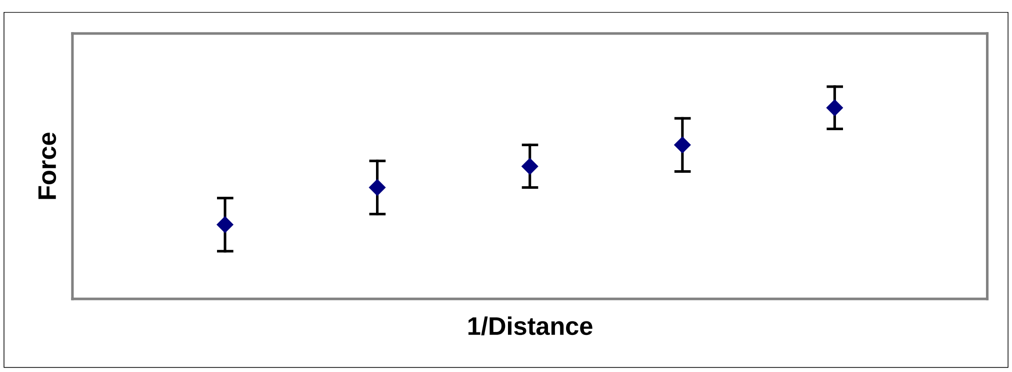
<u>Spring:</u>

12. If a spring stretches *5 cm* when you put *100 g* on it, what is the spring constant?

13. If you set a mass of 10.0 ± 0.1 g on a spring with spring constant, $k = 5.0 \pm 0.2$ N/m, how long will it stretch?

<u>Best-Fit Line</u>:

14. For the following force versus 1/distance diagram draw in the maximum and mini-



mum slope and the best-fit line.

15. Fill in the table and draw a force vs. stretch diagram for the following data! Draw the maximum and minimum slope and the best-fit line. Give the value of the spring constant including uncertainty (via maximum and minimum slope curve).

Stretch in cm	Force in N
10	1 ± 2
25	3 ± 1
50	4 ± 2
75	8 ± 1
100	9 ± 1

Explanations:

17. Explain why a line fit to many measurements gives a better result than a single measurement.

18. Try to explain, using forces and levers, why your biceps is such a strong muscle.

19. Put a string through a book with the binding facing upwards. Try to pull the strings until they are horizontal! Why is this impossible (your string will usually break)?

Experiment 1-3

Velocity, Acceleration, and *g*

1. Introduction

In this experiment you will study motion with constant velocity and motion with constant acceleration. Ordinarily, it is difficult to examine these kinds of motion with precision, since objects in free fall tend to move too rapidly, and frictional forces arise in most everyday situations. These factors hinder a direct observation of the underlying physical principles of motion, and in fact this is one of the reasons why these principles were poorly understood until Galileo's famous experiments. In this lab, it will be possible to study motion in the absence of almost any friction by using a rider on an air-track. The air-track has rows of small air jets running down its side, which support the rider on a thin film of air and allow it to float just above the track – minimizing contact and thereby minimizing friction. When the track is level and the rider is given a slight push, it will move with constant velocity; when the track is slightly inclined, the rider will experience a small acceleration due to the component of gravity which is parallel to the track.

To study the motion of the rider, you need to be able to make accurate measurements of its position at given intervals of time. This is done using a sonar device called the Sonic Ranger. This apparatus sends out discrete pulses of sound waves, which are reflected back by the object or objects in its path. The Sonic Ranger is in turn connected to a computer, which calculates the distance to the object based on the time it takes the signal to leave the sonar module and return. If a series of such measurements is made in rapid succession, then the computer can reconstruct the motion of the rider over some time interval, and this information can be used for other calculations, such as the "instantaneous" velocity or acceleration of the rider as a function of time. An essential part of this lab is becoming familiar with using the computer to obtain and analyze data. The computer offers many extraordinary advantages over manipulating data by hand, but you can benefit from it only if you have learned how to use it effectively and understand its limitations.

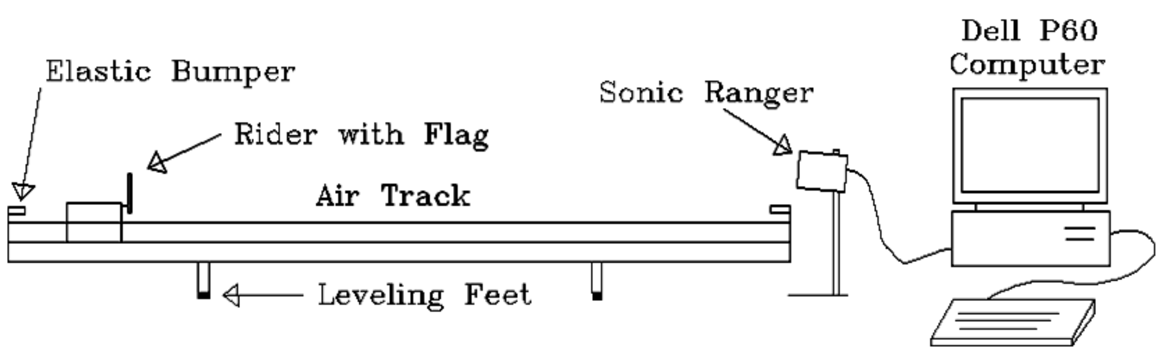


Figure 1.

2. Set-up

2.1 Getting Started

Some of the laboratory computers require you to log on before you can view and use the desktop screen. To log on, use the user name **student** and the password **student**. When the desktop screen is visible, double click on the VELOCITYLAB icon to load the Sonic Ranger software. (In this lab, all mouse clicks use the left button.) The screen shown in Figure 2, with two empty graphs displayed, should now appear. The computer is now ready to take data. The top graph will show position (meters) versus time (seconds), while the bottom graph will show velocity (meters/second) versus time (seconds).

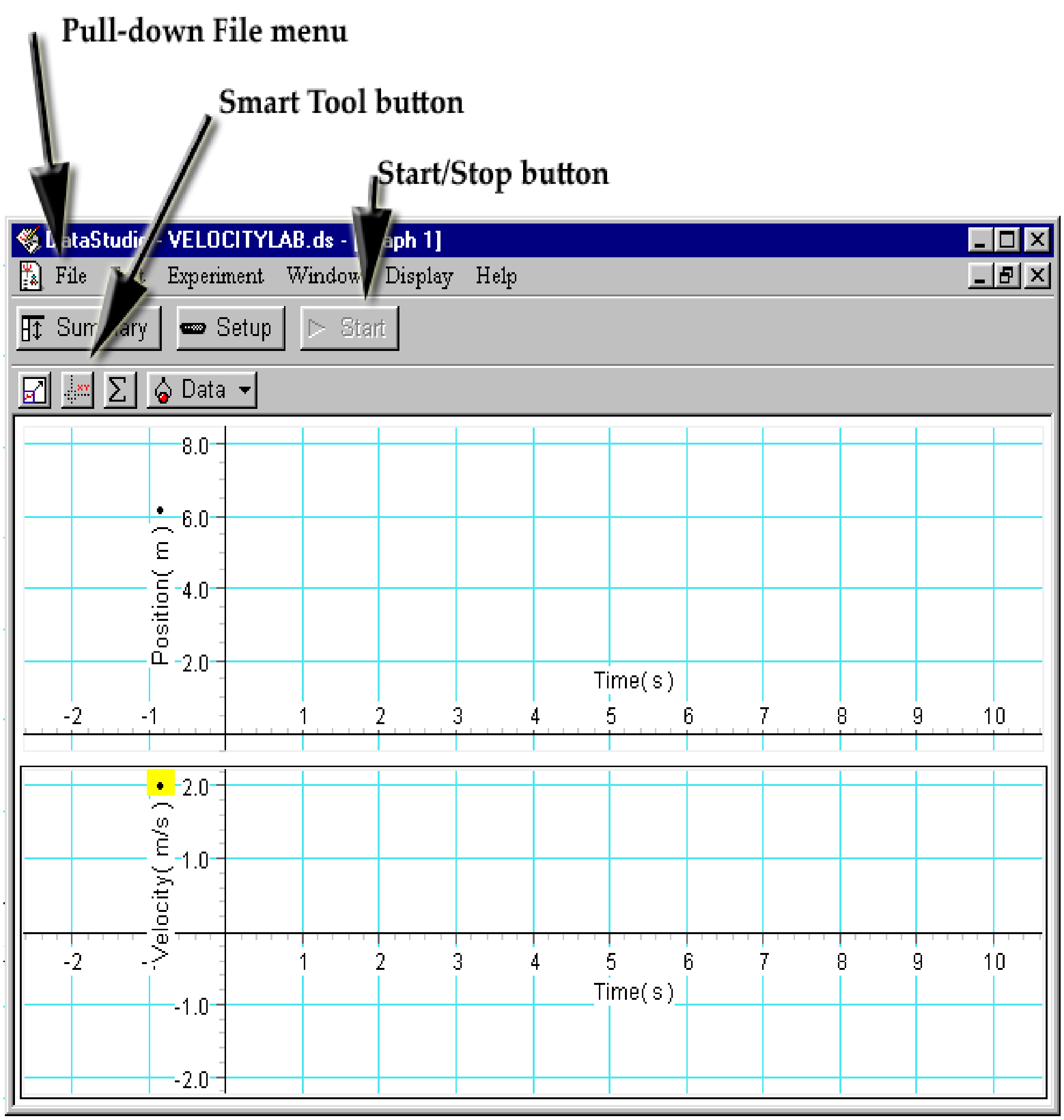


Figure 2.

To begin taking data, single click on the screen's **Start** button. As the data are being collected, both graphs will display the measurements. To stop taking data, single click on the screen's **Stop** button (the button that replaced the Start button). After analyzing the graphs, you can erase the data and begin collecting new data by single clicking on the **Start** button.

The Sonic Ranger measures and records the position of an object every 0.05 seconds. The graph of position versus time is then obtained directly from these position measurements. The software constructs the graph of velocity versus time by calculating the average velocity (the change in distance divided by the time interval) based on the successive position measurements. Since the position measurements are taken very closely together, the calculated average velocity for each interval is reasonably close to the instantaneous velocity at any instant within that interval, as long as the rate of change of the velocity is small.

The Sonic Ranger has two modes of operation, which are controlled by the switch on top of the Sonic Ranger module. Move the switch to the person icon to take good-quality data from a large object (such as a moving person) using a wide sonar beam. Move the switch to the cart icon to take good-quality data from a small object (such as a rider moving on an air track) using a narrowly focused sonar beam.

You can now begin taking data and displaying your results by trying the following simple experiment. Switch the Sonic Ranger to the person mode. Stand about six to eight feet away from the Sonic Ranger and have your lab partner point it toward you. When your partner single clicks the Start button to begin taking data, start moving slowly toward the Sonic Ranger. Try to see if you can move with constant velocity. Your partner can end the data collection after you have walked for about ten seconds. Can you tell from the graphs whether or not you have been reasonably successful in moving at constant velocity? You may have to try a few times.

Note that the position, velocity, and time axes will automatically rescale so that all of the data points are visible on the graphs. To change a graph scale manually, just click and drag any number on the appropriate axis. Clicking and dragging the axis line will move the axis in the display window. Notice that the time axes of both graphs will rescale and move together so that the time axes of the position and velocity graphs are always aligned.

Collect some more data by using a more complicated motion. After each data run, compare the distance and velocity graphs and be sure that you understand how the shape of the velocity graph can be predicted from the position graph. For each data run, you may have to manually rescale the position and/or velocity axes to have an appropriate display of the graph details. For example, a very small non-zero velocity might appear as zero meters/second if the velocity scale is not sufficiently expanded.

Before starting the next part of the experiment, you may want to reset your display settings to their original default values. To do this, simply quit the program by single clicking on the pull-down **File** menu and then choosing the **Quit** option. A dialog box will ask if you want to save the activity. Choose the **No** option and the screen will return to the desktop display. (You cannot save the activity, and choosing the **Yes** option or the **Cancel** option will just make the dialog box disappear without closing the program.) You can double click on the desktop's VELOCITYLAB icon to restart the program. You may follow this procedure whenever you want to clear the screen and return to the original default settings.

2.2 Setting Up the Sonic Ranger for Use with the Air Track

Set the Sonic Ranger just at the right end of the track, and switch the Sonic Ranger to the cart mode. Adjust the location and tilt of the device so that the sonar beam travels horizontally along the center line of the air track. This will allow you to make sure the Sonic Ranger is able to detect the rider when it is at the far end. Turn on the air for the air-track and set the rider at the far left end of the track. Prop up the right side of the track so that the rider stays at the left end. Follow the same procedure as you did before to collect data. The distance from the Sonic Ranger to the flag on the rider will be approximately 1.8 m, so if it is aimed correctly you should see a stationary object about 1.8 m away on the graph. If not, try to re-aim it while you are taking data or try taking data again. You can then check the alignment more precisely by giving the rider a small push along the air-track, and observe whether the computer plots the rider's position smoothly throughout the length of the track. If there are “static-like” jumps in the plot, then the sound waves are not reflecting properly back to the Sonic Ranger, and you will need to align it more carefully.

2.3 A Note about Distances

The Sonic Ranger is unable to detect objects that are too close, because it requires a certain amount of time between sending and receiving signals. Close objects will reflect signals too early for the Ranger to interpret the data correctly. Therefore**,** always work with distances greater than 20 cm from the Sonic Ranger to insure that the distance to the object is determined correctly**.**

Note also that the scale on the air-track increases as you read toward the right. Since the Sonic Ranger is aimed left, its scale increases toward the left, with the origin at the faceplate of the module, not the right end of the track (see Figure 3). It is extremely important that you keep these two scales separate in your work. You should use the readings from the Sonic Ranger (which the computer indicates in meters) for all of your experimental calculations, and use the scale on the air-track (which is marked in centimeters) only for positioning the rider in the same place when you are repeating an experiment over several trials.

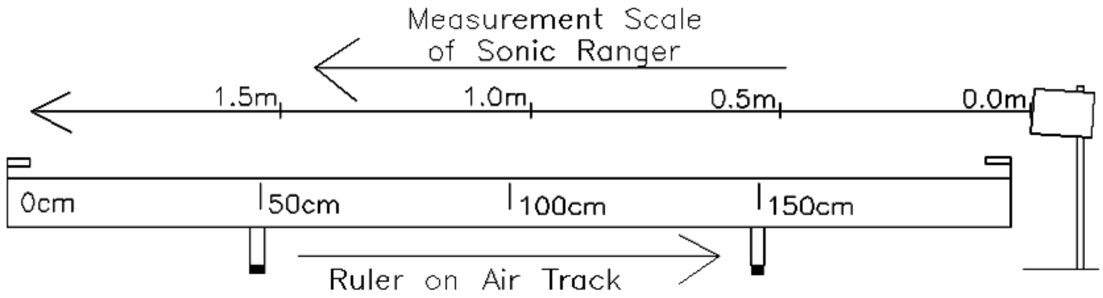


Figure 3. The measurement scale of the Sonic Ranger starts at its faceplate, and increases toward the left, *oppositely of the rider on the air track.*

2.4 A Note About the Rider and Air-Track

It is important that you take care when using the rider and air-track. Don't let the rider sit on the track when the air is off, and don't let the flag-side of the rider collide with the elastic bumper. Both of these can cause the rider and air-track to scrape against each other, leaving scratches which permanently damage the equipment.

<u>3. Motion with Constant Velocity</u>

3.1 Leveling the Air Track

To study motion with constant velocity it will be necessary to level the air-track as carefully as possible so that the rider does not tend to accelerate in one direction. The left side of the air-track has two adjustable feet; the right foot is not adjustable, but you may raise it by stacking sheets of paper beneath it. (Avoid using the metal shims for leveling the track, since you will be using them later to raise it to a fixed inclination.) The air-track has been machined to remain extremely straight along its entire length, but because the rider is also extremely sensitive to the slightest variation along the track, you will find that the rider sometimes remains stationary in one region of the track but tends to drift when it is in another region. It is not always possible to completely level the track, but you should try to minimize these irregularities by making sure that the drift is as small as possible in most parts of the track, and that the direction of the drifts are more or less random. (If all the drifts tend in the same direction, then this indicates that you can make the track more level.)

3.2 Taking Data

Once you are satisfied that the track is sufficiently level, set the rider at the 150 cm mark. Begin collecting data, and then give the rider a gentle push toward the left. Make sure that you are able to take data over a substantial portion of the return trip after it has bounced off the elastic bumper at the left end, and also make sure that the position data is smooth and without jumps.

- Is the velocity graph what you expected? The time scales of the two graphs are always the same, so you should be able to see how the rate of change in the displacement graph corresponds to the velocity.
- How does motion with constant velocity on the air track compare with trying to walk with constant velocity?

3.3 The Coefficient of Restitution

When two objects collide and bounce away from each other, they tend to lose some of their energy in the collision, and the rebound velocity between the two objects is therefore less than the initial velocity between them. This is why objects that are dropped will sooner or later stop bouncing. The elasticity of the collision can be indicated by *e*, **the coefficient of restitution**, which is defined as the speed after the collision divided by the speed before the collision:

$$e = \left| v_f \; / \; v_i \right|$$

A perfectly elastic collision, one in which no energy is lost, would therefore have a coefficient of restitution equal to one; an elastic "super" ball is a good example of an object whose coefficient of restitution in many collisions is often close to one. You can calculate *e* for the case of the rider colliding with the elastic bumper by using the data you collected in this experiment.

3.4 Using the Smart Tool to Read Off Data Points

To use the Smart Tool in one of the graphs; select a graph (position or velocity) by clicking in the first quadrant of the graph and then clicking on the menu's Smart Tool button (this should be the second square button on the third line of icons). This will activate crosshairs that can be used to select a data point and display its coordinate data pair. As you get closer to a data point, the Smart Tool will "gravitate" toward that data point. To change the position of the crosshairs of the Smart Tool, move the cursor over the center of the Smart Tool until the pointer turns into two crossed double arrowheads and a hand. Drag the cross hairs of the Smart Tool to the desired location. To deactivate the Smart Tool in a graph that has been selected, just click the Smart Tool menu button again.

Note that if you activate the Smart Tool in the second graph before the Smart Tool has been deactivated in the first graph, cross hairs will appear in both graphs simultaneously and can be moved together along the common time axis.

3.5 Determining the Coefficient of Restitution

You should calculate *e* from the data just collected using two different methods.

In the first method, use the position versus time graph with the Smart Tool to read off and record the (t,x) coordinates of 5 reasonably separated points before the bumper collision and 5 reasonably separated points after the collision. Choose representative points that are not part of extraneous effects (which occur at the ends of the motion, in the small region of the collision, and at irregularities in the air track itself). On graph paper; plot the (t,x) coordinates of the 5 points before the collision, draw a best fit straight line through the 5 points, and then determine the initial velocity from the slope of this line. Do the same for the 5 points after the collision and determine the final velocity.

- Calculate *e* from the ratio of the two velocities.

In the second method, use the velocity versus time graph with the Smart Tool to read off and record the velocity values for 5 reasonably separated points before the collision and 5 reasonably separated points after the collision. Once again, choose representative points that are not part of extraneous effects. Average the 5 values before the collision and then average the 5 values after the collision.

- Calculate e from the ratio of these two average values.
- How would you explain any differences between the results from the two different ways of calculating e? Which method do you think gives a more accurate value for *e*?

4. Gravitational Acceleration

A small constant force can be applied to the rider by inclining the track slightly. The component of gravity which acts on the rider parallel to the air-track is equal to $g\sin\theta$, as indicated in Figure 5.

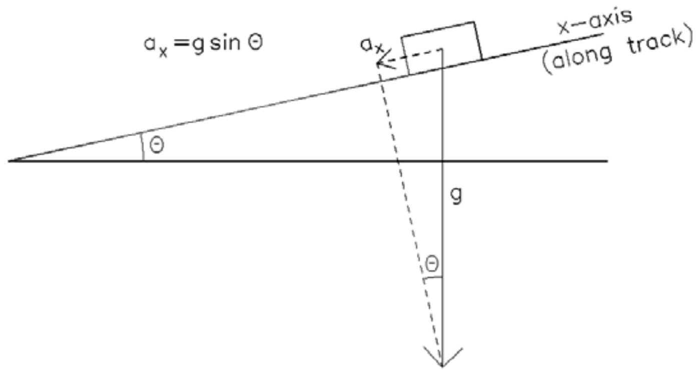


Figure 5.

Use a few of the shims provided to elevate the right side of the air-track. NOTE: All of the shims are 1.24mm thick.

Note that $\sin\theta$ equals the height of the shims divided by the length of the track between the supports, which in this experiment is exactly 1.0 m (see Figure 6) so $\sin\theta$ is equal to the height of the shims.

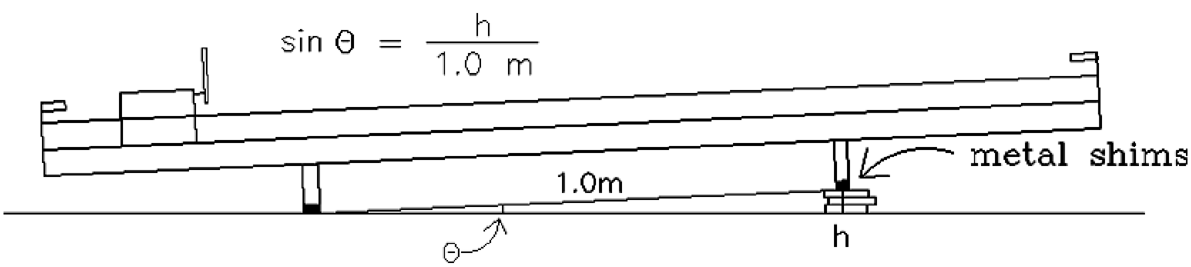


Figure 6.

A convenient method for taking data is as follows. Set the rider on the air track at a given point, say at the 150 cm mark, and release it. Start taking data and record the motion as the rider goes down the track, bounces off the elastic bumper, goes back up the track, and then heads back down the track.

You should print out the resulting graphs on the screen to include in your lab report. To print: click on the **File** menu, then choose the **Print** option, and finally select **OK** in the dialog box. The laboratory's network printer is located on the side of the room (printouts are labeled according to the number on your setup's computer).

You can calculate the acceleration along the air track from the data displayed on the computer screen. Using the Smart Tool simultaneously in the position and the velocity graphs, read off and record the (t,x) coordinates and the (t,v) coordinates at 5 different times during the upward-downward motion that occurred after the initial bumper collision. Choose 5 representative points that are reasonably separated and are not part of extraneous effects (which occur at the ends of the motion, in the small region of the collision, and at irregularities in the air track itself). For each point of time chosen, be sure to use the x value and the v value at the same value of the time (to an accuracy of about 0.025 seconds). The acceleration along the track can now be calculated in two different ways.

In the first method, use the equation

$$x = x_0 + v_{x0}(\Delta t) + \tfrac{1}{2} a_x (\Delta t)^2$$

Solve for a_x, and then calculate four values for a_x by using the data for four different time intervals. Find the average of a_x for this method. What value does this a_x give for g?

In the second method, use the equation

$$v_x = v_{x0} + a_x t$$

On graph paper; plot the (t,v) coordinates of the 5 times chosen, draw a best straight-line fit to the 5 points, and then determine the a_x from the slope of this line. What value does this a_x give for g?

- Are your values for g close to the accepted value?

- Discuss possible sources of error in each of the two methods.

Experiment 1-4

Geometrical Optics I: Introduction

1. Introduction

This experiment is the first in a sequence of three experiments introducing the basic ideas of geometrical optics. These experiments are unrelated to the material you will cover in class this semester.[1] They are designed to present the ideas of geometrical optics from an empirical point of view.

In this first experiment, we utilize a simple model describing light as a bundle of rays. We then explore the behavior of these rays as they reflect from smooth surfaces (mirrors) and as they are transmitted, or refracted, through transparent media. We experimentally verify Snell's law of refraction and observe the phenomenon of total internal reflection as a consequence of Snell's law.

Why is optics important (besides the fact that the human eye works by the rules we describe here)? Optical instruments are used in many real-life situations in which an image of a system is needed. Such cases apply whether the system is large or small, and whether easily accessible or not. For example, whenever you want insight into how a biological system works, you will likely need to use imaging methods based on geometrical optics. Even if you are only interested in the final data from some fancy imaging system, it will be essential for the quality of your work that you know how to evaluate what you are seeing and what the limitations of the method are.

Remark: It is strongly recommended that you look over the setup in the Lab Library before coming to the lab. This lab has many subparts and getting stuck at one point may make it difficult to finish the lab. It is difficult to understand optical phenomena in the abstract so simply reading the lab manual is insufficient preparation for this lab. Be sure to stop by the Lab Library and let the TA show you the apparatus!

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[1] The material in these labs is discussed in the assigned lecture course textbook (Fundamentals of Physics, 8th Ed., Halliday, Resnick & Walker, Chapters 34-38). Next semester, you will study topics in both geometrical and physical optics in lecture, which include interference and diffraction phenomena. Both geometrical and physical optics are understood as direct consequences of the wave nature of light. It is the "wavelength" of the light waves which is responsible for the different colors that we see (see section 2.3).

2. Theory

2.1 Geometrical Optics

Geometrical optics is a simplified way of describing light phenomena. It is valid as long as we do not consider cases in which light passes through small pinholes or slits, or examine the edges of shadows. In the second semester lecture and lab, you will encounter effects such as diffraction and interference, which cannot be explained by the theory of geometrical optics. The assumption for geometrical optics is that rays of light propagate along straight lines until they are reflected, refracted, or absorbed at a surface.

A very simple, but important, application of geometrical optics is X-rays. (X-rays are a type of 'light', with wavelengths that are much shorter than those characteristic of visible light.) The shadow image of the skeleton made in an X-ray may be understood by simply propagating straight lines from the source to the detector, except for those rays absorbed by some tissue (e.g. bone) in the path.

2.2 Light as Rays

The concept of light rays may become more plausible to you if you imagine being in a dark, dusty room with light from the outside entering through a small hole. You can "see" a ray of light traveling in a straight line from the hole to wherever it hits the wall. (Your eye actually sees the light scattered by dust particles along the path of the ray.)

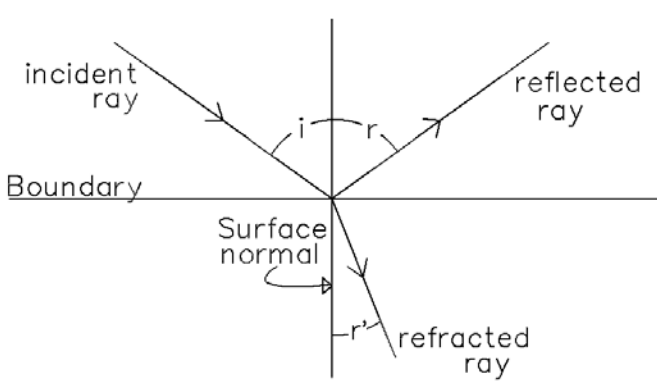
We describe light using the following simple model. A light source emits rays of light in all possible directions. Each ray propagates along a straight line until one of the following happens. It is:

(1) reflected if it encounters a reflective material.

(2) refracted if it encounters a transparent material.

(3) absorbed if it encounters a non-reflecting, non-transparent material.

For most materials in the real world, combinations of these effects occur.



In geometrical optics, we assume that each ray of light travels along a straight line indefinitely unless it strikes a boundary, like a mirror or an interface between air and another material. A source of light, such as a flashlight, produces many rays that travel nearly parallel to each other on their way to the illuminated spot on the boundary. If a ray is not absorbed when it strikes the boundary, it splits into parts as shown in the figure above. Both the reflected and refracted rays again travel in straight lines until they encounter another boundary. The angles i, r, and r' are measured with respect to the normal (perpendicular) to the boundary surface.

Although in actuality a light ray that encounters a surface experiences all three of these effects, there is usually one that dominates. If the boundary is a metallic mirror, only the reflected ray is relevant, and if the boundary is a transparent medium such as water, only the refracted ray is relevant. In sections 2.4 and 2.6, we describe the quantitative relationships between the incident ray and the reflected and refracted rays. In particular, we write the relations between the angles i and r, and between the angles i and r' shown in the figure.

2.3 Absorption/Color (a brief aside)

The colors we perceive in light are directly correlated with the wavelength of the light ray (for example, blue light has a shorter wavelength than red light). White light is the response your brain perceives if light of all colors strikes your eye’s receptors.

Materials that absorb light can either decrease the reflected intensity for all colors equally (which means that the light simply appears less bright than before) or they can selectively decrease the intensity of some colors. An example of the second case is white light from the sun falling onto the grass where the grass absorbs all colors except green (which it reflects). This is why grass appears to you as green. If an object absorbs all the light, it appears to be black.

All materials absorb to some extent, even when the light appears to pass through or reflects. (The best commercial mirrors reflect about 99.99% of the incoming light.)

2.4 Reflection

The ray reflected from a surface emerges at an angle equal to the angle of incidence of the original light ray. Quantitatively, this law of reflection is expressed in terms of the angles shown in the figure as:

$$r = i$$

There are two general types of reflection - diffuse and specular. Diffuse reflection is exemplified by the reflection from the surface of this page. On a microscopic scale, the surface of the paper is quite rough; consequently, parallel rays striking even nearby parts of the paper's surface are characterized by different angles of incidence. Each of the initially parallel rays is reflected in a different direction. Therefore, when we shine a flashlight on a piece of paper, irrespective of our position, we see a bright spot on the paper.[2] In this case, it is not feasible to determine the relationship between *i* and *r* for any particular ray.

If parallel rays reflect from a surface which is very flat and smooth (such as that found on a mirror or on window glass), there is a unique angle of incidence and therefore a unique angle of reflection. This type of reflection is referred to as specular or regular. A flashlight beam reflected specularly will only be observed if it is viewed along the direction of reflection. We restrict our attention here to specular reflection, since only in that case the relationship between the incident and reflected rays can be understood well enough to be used in optical instruments.

You may have seen fresh snow glitter in the sunlight, and wonder which type of reflection that is. The explanation is that some of the small flat surfaces of the snowflake are smooth and act like mirrors that reflect several rays in the same direction, which just happens to be where your eye is. But the totality of the surface of white snow is irregular and reflection from the snow more often looks much like the diffuse reflection from this page.

Remark for Experts: There is of course no perfectly smooth surface in nature. Every surface is rough on the atomic level. To get specular reflection, it is sufficient that surface irregularities are small compared to the wavelength of light. So when you polish anything, from optical instruments to your furniture, the desired successful appearance is accomplished when you have made all surface irregularities small compared to the wavelength of light (which is about 5×10^{-5} cm, or 500 nm). Since atoms are much, much smaller than that, there is no contradiction.

2.5 Real and Virtual Images

When you look in a mirror, you see your own image. It looks as if a copy of you is standing behind the mirror. But if you put a screen behind the image at the position where your copy seems to stand, you would obviously not capture the image of yourself on the paper. Your copy in the mirror or any image you cannot capture on a screen at its apparent location is called a virtual image. If you can record the image on a screen at its apparent location, it is called a real image. Real images are caused by the actual convergence of light rays at a given point; virtual images are caused by light rays that diverge in such a way that they seem to, but do not actually, converge at a certain point

Mirrors produce virtual images. In the next two labs, we will deal with many different examples of real and virtual images.

[2] Only a few of the light rays are reflected into our eyes!

2.6 Refraction and Snell's Law

Light waves (rays) propagate through vacuum with a fixed velocity *c* equal to about 300,000 km/s or $3 \cdot 10^8$ m/s. One of the consequences of our understanding of electromagnetism, detailed in Einstein's theory of special relativity, is that nothing can travel faster than this speed.

As the waves travel through any material, interactions of the light with the atoms result in a velocity *v* which is smaller than *c*. The ratio of these speeds is called the refractive index *n*, and is a specific constant associated with the medium:

$$n = \frac{c}{v}$$

For visible light traversing through most transparent media, the index varies roughly between 1 and 2.5, depending on the material. Although in actuality, light travels through air slightly slower than it does through a vacuum, for our purposes we will consider the refractive index to be 1. Typical glass, for example, has a refractive index of about 1.5.

When a ray of light in air encounters a medium, the change in wave velocity requires it to change direction.[3] The new angle relative to the normal, shown in the figure, is the angle of refraction (*r'*) given by Snell's Law:

$$n_{r'} \sin r' = n_i \sin i$$

which reduces to the following equation when the incident ray travels through vacuum/air.

$$\sin r' = \frac{\sin i}{n}$$

For example, if an incoming ray at an angle $i = 15°$ encounters glass (with an refractive index of $n = 1.5$), then the refracted ray will change direction and travel through the glass at an angle of $r' = 9.9°$. Since *n* is always larger than unity, the refracted ray entering a material is closer to the normal than the incident ray.

2.7 Total Internal Reflection and Critical Angle

Snell's Law describes the paths of light rays traversing interfaces between any two media. It is equally valuable for analyzing light rays traveling from air to another medium or vice versa. If we reverse the directions of the arrows and the labeling of *i* and *r'*, our picture in the figure of section 2.2 is still a correct description of the path of the light ray. We can therefore see that when light passes from glass to air, the ray exits with a larger angle to the normal than the incident ray. In this example Snell's Law can be reduced to:

[3] The directional change follows from a simple physical argument. This will be discussed next semester.

$n \sin i = \sin r'$. The refracted ray must have a larger angle to the normal than the incident ray, but this angle must always be less than 90° in order for the refracted ray to emerge on the opposite side of the interface from the incident ray. Therefore, at some maximum incident angle the refracted ray travels along the surface; for angles beyond this, no refracted ray emerges, and the ray is totally_reflected back into the glass. This angle is called the critical angle, θ_c, and from Snell's Law is defined by:

$$\sin\theta_c = \frac{1}{n}$$

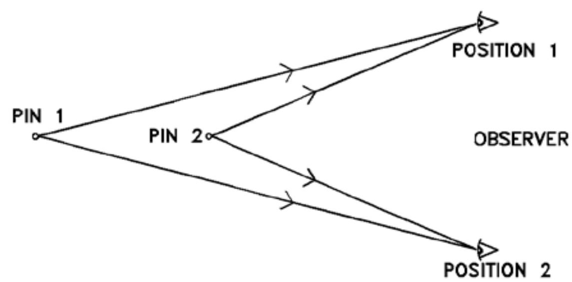
An easy way to determine the index of refraction of a medium is by measuring the critical angle and using this equation. As you can see, total internal reflection can only happen when a light beam passes from a medium with a higher refractive index to a medium with a lower refractive index.

2.8 Method of Parallax

In this experiment you will be analyzing the reflection and refraction of light rays by determining the location of virtual images. Virtual images are difficult to locate as they cannot be captured on a screen.

Imagine riding in a car and looking out the window at a forest on the side of the road. If the car was stopped, you could try to line up two trees in your line of vision. You might be temporarily successful, but as soon as the car would start moving again, those two trees would no longer be lined up because of the shift in your perspective. You could probably convince yourself that only if two trees were at the same location could they stay lined up in your field of vision even if your location changed.

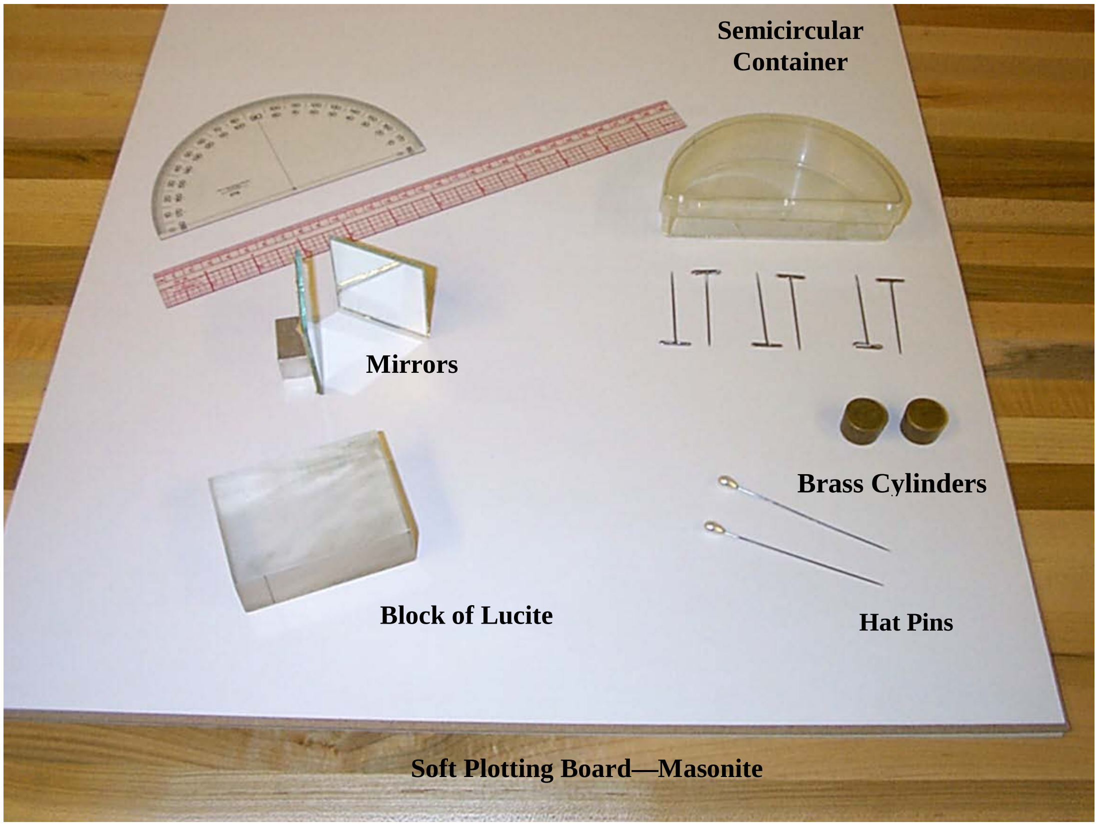
In the figure at right, the observer at position 1 sees pin 2 to the left of pin 1. As the observer moves to position 2, however, pin 2 appears to move over to the right of pin 1. These apparent motions, parallax, are only removed if pins 1 and 2 are at the same location



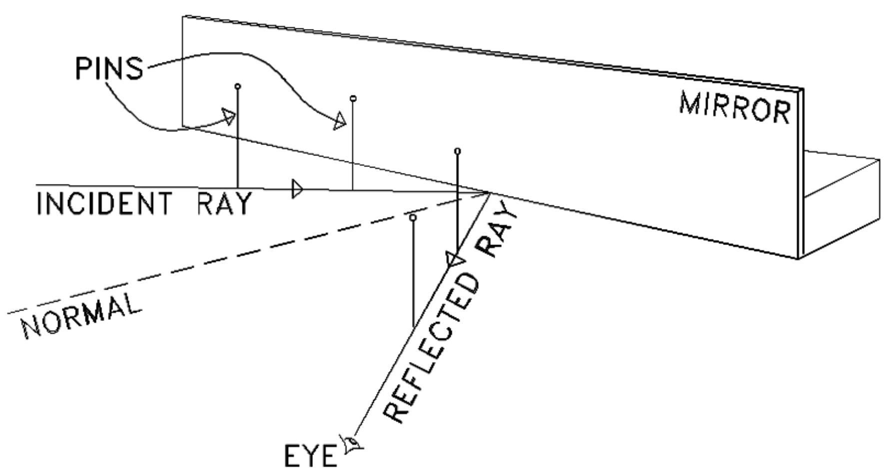
This is the concept we will use to determine the location of virtual images. Although you cannot capture the virtual image on a screen, you can compare its location in our line of vision to the location of an object, called a "finder pin." If two objects remain lined up even when your perspective changes, you can conclude, based on the previous discussion, that they are at the same location.

Therefore, you will determine the location of the finder pin where it and the image are lined up irrespective of your line of vision, and mark that as the location of the virtual image.

3. Experiments



3.1 Reflection



In the first part of the lab, we verify the law of reflection with a mirror. Mark the location of the mirror by drawing a line along the back edge of the mirror (where the reflection takes place). Like a line, a ray can be specified by defining two points. We specify a ray using two pins placed in front of a mirror. The two pins form a line; when your head is placed behind the pins so that you see them as a single pin, you are looking along the direction of the incoming ray. If you now move your head to the other side of the normal, you will find these two pins to lie on top of each other only when your eye is at the location of the reflected ray. Mark this reflected ray by two new pins placed so that they are lined up with the other two pins along this line of sight. You should have four pins (two on each side of the mirror normal) that are exactly on top of each other when viewed along either the incident or reflected directions. Remove the mirror and the pins from the paper and draw the incoming and reflected rays using the pinholes in the paper and also the normal to the mirror through the intersection vertex of the two rays.

- Do the two rays intersect at the reflecting edge of the mirror?
- Measure the incident and reflected angles relative to the normal.
- Estimate a reasonable uncertainty for these angles, considering the technique used to construct and measure them.
- Are the two angles equal within uncertainty?
- What were the main errors performing this experiment?

3.2 Image from a plane mirror

In the second part, we find the virtual image of a mirror using two different methods. Mark the location of the mirror on a new sheet of paper. Place a pin in front of the mirror and put a second pin behind the mirror at the position at which there is no parallax between it and the original pin. The pin behind the mirror appears atop the pin in the mirror and remains there even if you move your head. Mark the position of both pins.

- Is the position found by the absence of parallax more accurate when you hold your head near to the mirror or far away?

The second method for locating the virtual image requires that we mark (with pins) two different reflected rays of the first pin in front of the mirror. This is done using the same method as in the previous section. The mirror and pins are then removed from the paper and we extend these rays behind the mirror and mark their intersection as the location of the virtual image.

- Does it coincide with the position found using the parallax free technique?

Now draw the lines connecting the intercepts of the reflected rays with the mirror plane and the initial pin in front of the mirror. This line shows the actual path the light took as it was reflected in the mirror.

- Give a simple geometric argument why the distance between the object and the mirror should exactly equal the distance between the virtual image and the mirror.
- What are the major sources of error in this part of the lab?

3.3 Multiple Reflections

Two mirrors produce multiple reflections. We will investigate two different configurations and analyze the reflections produced.

First, two mirrors are placed at a 90-degree angle between them and their reflecting surfaces facing in. An object placed between the two mirrors should produce <u>three</u> images: one image from each mirror, plus an additional image between them, which is an image of an image, produced by reflections from both mirrors. Write your name on a small piece of paper and hold it up to the mirrors.

- How many images do you see?
- Your name could appear written backward or forward. Note which images appear which way.
- Explain why your name appears written forward in the middle image.

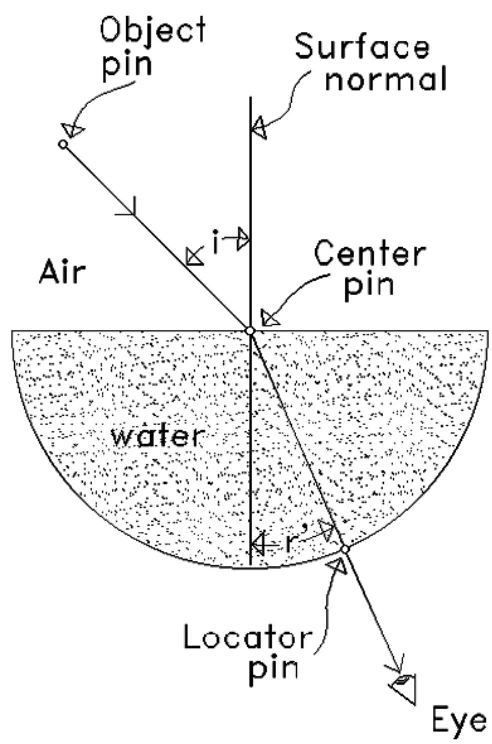
Something very different occurs if you place the two mirrors parallel and facing each other. When you place a pin between them, you see a series of pin images in the mirror due to multiple reflections in the mirrors.

- Explain why you see several images of the pin.
- Why do the more distant images appear dimmer?

3.4 Snell's Law

In this part, we verify Snell's Law and use it to measure the refractive index of water. The picture at right shows the setup of the experiment. Fill the semicircular container with water and place it on a new piece of paper. Trace the flat edge of the box on the paper, and mark the paper at the exact center of the flat surface with a pin.

Place an object pin on the paper on the side of the flat edge of the container. Look through the water from the curved side of the box. Place a locator pin so that it appears lined up with the other two pins (object pin and center pin).

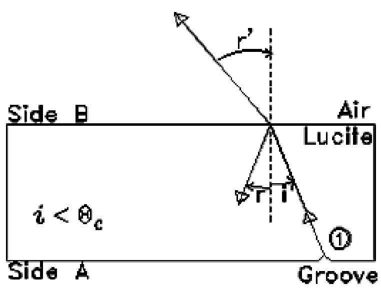


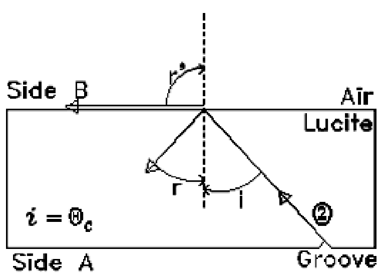
Repeat this procedure until you have a total of 5 pairs of rays on the same diagram. (Be sure to label which refracted ray goes with which image.) Remove everything and draw the rays using the pinholes. Draw the normal to the edge of the box through the center pin. Measure the angles, relative to the normal, of all the incoming and refracted rays and plot sin *i* vs. sin *r'*.

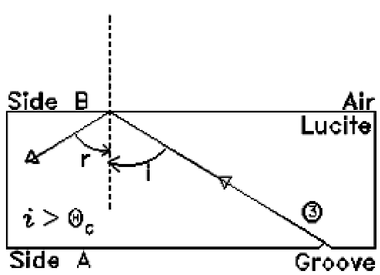
- Explain why your data points should now appear on a straight line.
- Draw a best-fit line. What are the slope and intercept (and associated uncertainties)? What should the intercept be? Determine the refractive index of water from the graph.
- Calculate the refractive index explicitly from each pair of rays in your data. Calculate the mean and estimate the uncertainty of a single measurement of the refractive index.
- The accepted value for the refractive index of water is about 1.33. Does the book value fall within one or two uncertainties of the mean?
- Note the main sources of error.

3.5 Total Internal Reflection and Critical Angle

Finally, we obtain the refractive index of Plexiglas (a form of transparent plastic called Lucite in Europe) by measuring the critical angle at a planar interface with air. A Plexiglas block with a small black groove is used as shown in the figures below. As noted earlier, light rays that encounter various materials can exhibit reflection, refraction, absorption, or a combination. In this part of the experiment, we will determine the critical angle of the interface between air and Plexiglas. The critical angle is the maximum incident angle of a light ray for which refraction occurs at a given interface. When light is incident at larger angles, it is no longer refracted and instead experiences "total internal reflection."







In the top figure, ray 1 is partially reflected back into the Plexiglas and partially refracted into the air.

In the middle figure, ray 2 is partially reflected back into the Plexiglas and -- in this special case -- is partially refracted into the air parallel to the surface.

In the bottom figure, ray 3 is incident at an angle, which is too large for it to be refracted; it is totally reflected.

When you view the top of the block from different angles, you see the black mark at different positions. If you now look along the Plexiglas surface of Side B, as shown in the middle figure, and mark the ray location alongside B, the angle between this point and the groove is the critical angle. From this, one can calculate the refractive index for the Plexiglas.

Take the Plexiglas and look along the top surface as parallel to the surface as feasible.

Measure the angle between the position of the groove and where you see it on the top surface relative to the normal. (This can be tricky! You may wish to lay one of the pins horizontally on the top surface to mark where you see the image of the black groove.)

- Determine the critical angle and use it to calculate the refractive index of the Plexiglas.
- How well can you repeat this measurement?
- Give the major sources of error!

4. Applications

Swimming pool experience:

The next time you go swimming, do the following experiment (safely!):

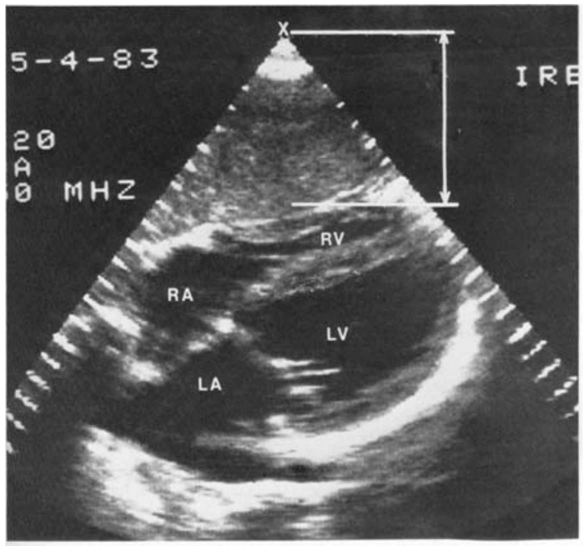
From underwater, look at people or trees outside the water. You will see that you can see them when you view with your eyes at small angles relative to the normal to the water surface. But as you look at larger angles, you find that the water surface suddenly appears weird, somewhat like liquid mercury. This occurs because the water surface reflects like a mirror so that you are not able to observe anything outside the pool. This is a consequence of total internal reflection. (If you will not be swimming for a while, you can also check this by looking at the water-air surface of a fish tank from below.)

Medical diagnoses:

A standard method of medical imaging utilizes ultrasonic sound echocardiography: ultrasonic sound waves produce images of inner body organs (along with many other applications). A wave of ultrasonic sound sent into the body is refracted and reflected as it passes through parts of the body of different density. The wave reflected from a specific organ is observed by a detector and produces an image. The manner in which the ultrasonic sound waves get reflected and refracted is similar to the case for light that we have considered.

An important difference between light and sound waves is that sound waves propagate faster in water than in air. (With light, it is the other way around!) This means that, for sound, water has a lower refractive index than air! Therefore, total internal reflection occurs going from air to water. To enable the sound waves to get into the body requires having no air between the source and the body, usually accomplished by putting a layer of gel between the sound head and the body. This also means you cannot send ultrasonic sound through air-filled organs (like the lungs).

The picture below shows an echocardiogram of a human heart:



LA = left atrium, LV = left ventricle, RA = right atrium, RV = right ventricle.

(Picture from: Marvin Berger: Doppler Echocardiography in Heart Disease.)

5. Lab Preparation Problems

Absorption:

1. How many times does a ray of light get reflected on a parallel pair of mirrors (which reflect 99% of the incoming light) such that the total intensity is down to 50%?

Reflection:

2. Explain in your own words the difference between specular and diffuse reflection using the following system and observations:

   On a day with no wind, the surface of the sea can be very smooth and shiny. On the other hand, the tops of the waves appear white in a strong storm.

Refractive index and Snell's Law:

3. What is the refractive index if a medium can slow down light to $v = 2/3\ c$?

4. If you measure an incoming ray from the air to be at an angle of 15 degrees to the normal, what angle will it have in the medium with an refractive index of $n = 1.2$?

5. If you measure $i = 25$ degrees (air) and $r' = 20$ degrees, what is the refractive index of the medium in which you measured r'?

6. Given the following pairs of values for i and r', draw a graph of sin r' vs. sin i. Determine the value of n using only the slope of the graph and not the individual values from the table.

i in degrees	0	15	30	45	60
r' in degrees	0	11.5	22.6	33.0	41.8

7. Given the following measured values for the refractive index, what is the mean and uncertainty (using the 2/3 rule)?

n=	1.24	1.21	1.29	1.23	1.27	1.26

Total Internal Reflection and Critical Angle:

8. Given an refractive index of diamond $n = 2.42$, what is its critical angle?

9. You want to measure the refractive index of water. For that you take a glass of water or go to your fish tank and look upwards at the air-water boundary through the side of your glass. When look at the boundary with a small enough angle, the surface will be (almost) 100 % reflective and you cannot see out through the boundary. With this setup, you measure the critical angle to be 60 degrees. What does this data give for the refractive index of water? Is it the same as the value you find in books?

Experiment 1-5

Geometrical Optics II: Thin Lenses

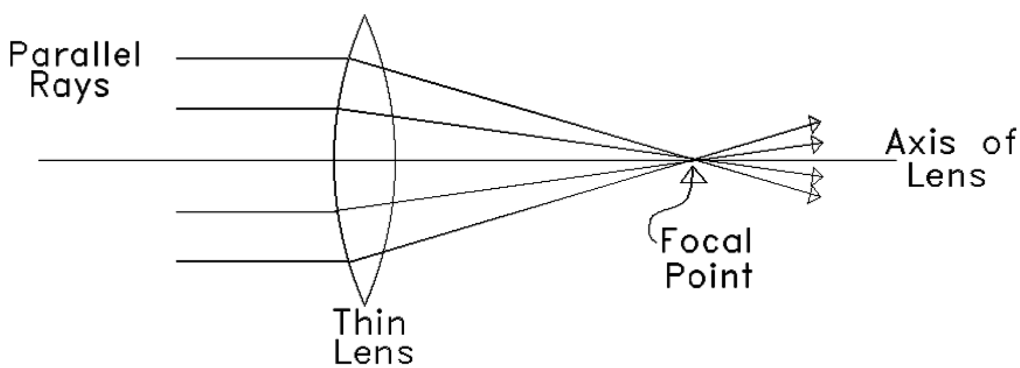
1. Introduction

Using the concept of light rays introduced in the last experiment, we will use it to describe ray diagrams and derive the lens equation.[1] These are powerful tools to draw conclusions based on geometrical optics. In this experiment, we will verify the lens equation for real and virtual images. In the next experiment, we will use these tools to build a magnifying glass, microscope, and telescope.

2. Theory

2.1 Lenses

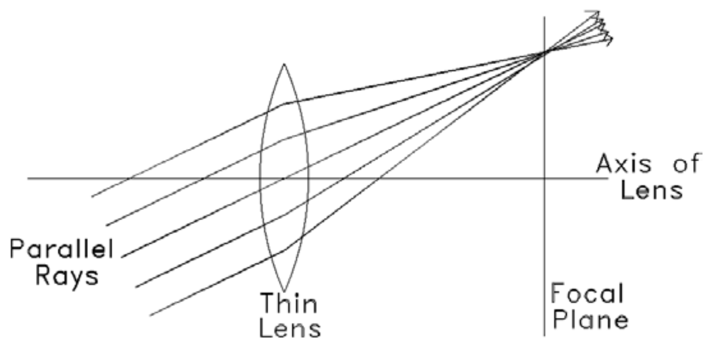
Lenses are made of glass, or a similar transparent material which refracts light at its surfaces. They are shaped so that they cause a bundle of parallel rays to converge to or (seem to) diverge from, a single point, called the focal point. The focal length, f , is defined as the distance between the lens and the focal point. The focal length is a fixed characteristic of a given lens, and depends only on the lens material and shape. A lens that converges light is called convex, and its value of f is positive. A lens that diverges light is called concave, and its value of f is negative. We deal only with converging lenses in this lab.



Remark: Sometimes lenses have a number printed on them which indicates the focal length, in millimeters. A positive number indicates a convex/converging lens while a negative number indicates a concave/diverging lens.

[1] Many approximations are made with "thin lenses". However, even the most sophisticated treatments of optics usually begin with these "thin lens approximations".

Remark for Experts: If a bundle of parallel rays falls on the lens but not along the axis of the lens, then the focal point will be shifted upward or downward along a line perpendicular to the axis (in the "focal plane"). Note that the focal distance along the axis stays the same.

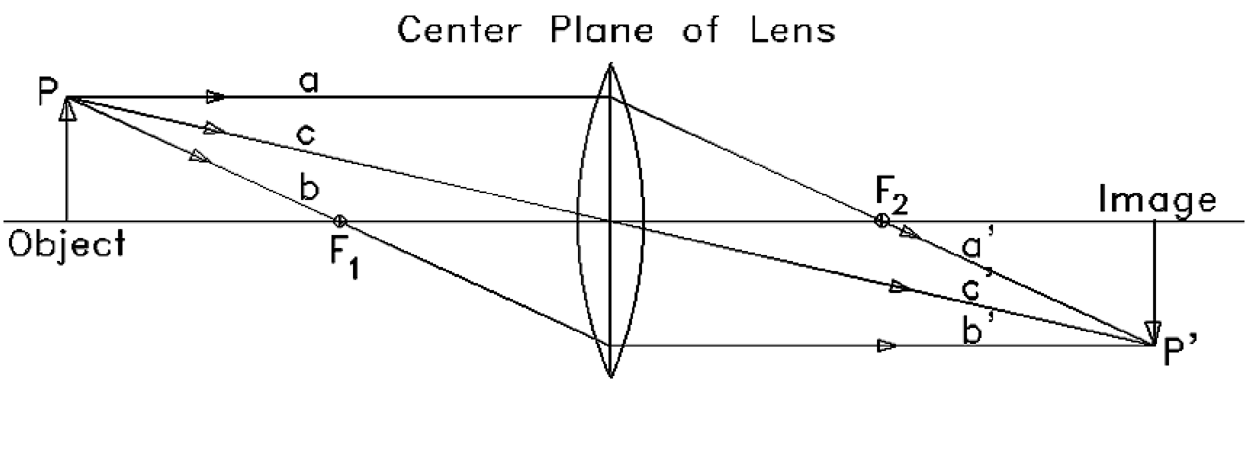


2.2 Ray Diagrams

Ray diagrams allow us to trace the paths of rays. They make it easier to understand how images are formed and what lenses do.

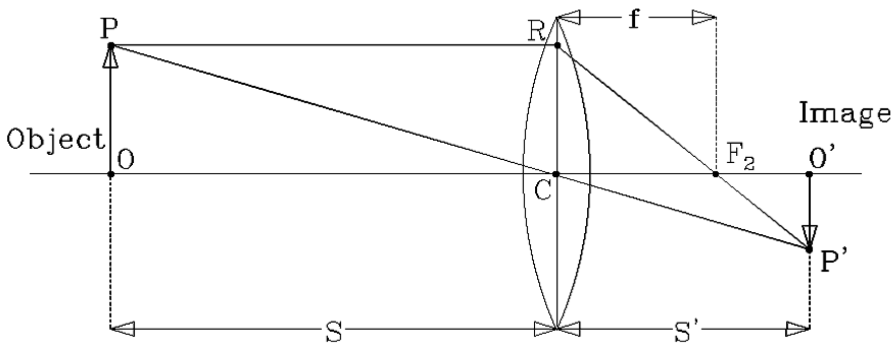
To draw ray diagrams, follow a few simple rules illustrated in the figure below:

1. Mark the focal points of the lens on each side of the lens.
2. If two rays from a source point intersect at another point, then all rays from that source point will intersect at the second point.
3. When drawing ray diagrams for convex lenses, it is most convenient to use three specific kinds of rays from the tip of the object (P) labeled a, b, and c in the figure and described here:
   (a) Rays that enter the lens parallel to the axis pass through the focal point behind the lens.
   (b) Rays that pass through the focal point in front of the lens leave the lens parallel to the axis of the lens.
   (c) Rays going through the center of a lens will go through the lens in a straight line and do not bend. (These are the only rays that have no net refraction from the lens)



2.3 Lens Equation

The ray diagram described in the previous section provides a graphical method for locating images. Using the simple geometrical argument given here, with the notation indicated in the figure below, we derive the lens equation. This equation relates the image distance *S′*, the object distance *S*, and the focal length *f*.



Since triangle *COP* is similar to triangle *CO'P'*:

$$\frac{S'}{S} = \frac{CO'}{CO} = \frac{O'P'}{OP} \tag{1}$$

Since triangle $F_2O'P'$ is similar to triangle F_2CR:

$$\frac{S'-f}{f} = \frac{O'F_2}{CF_2} = \frac{O'P'}{RC} = \frac{O'P'}{OP} \tag{2}$$

Combining equations (1) and (2):

$$\frac{S'}{S} = \frac{S'-f}{f}$$

which reduces to:

$$S'f = SS' - Sf$$

and finally, dividing by *f S S′* and rearranging, we obtain the thin lens equation:

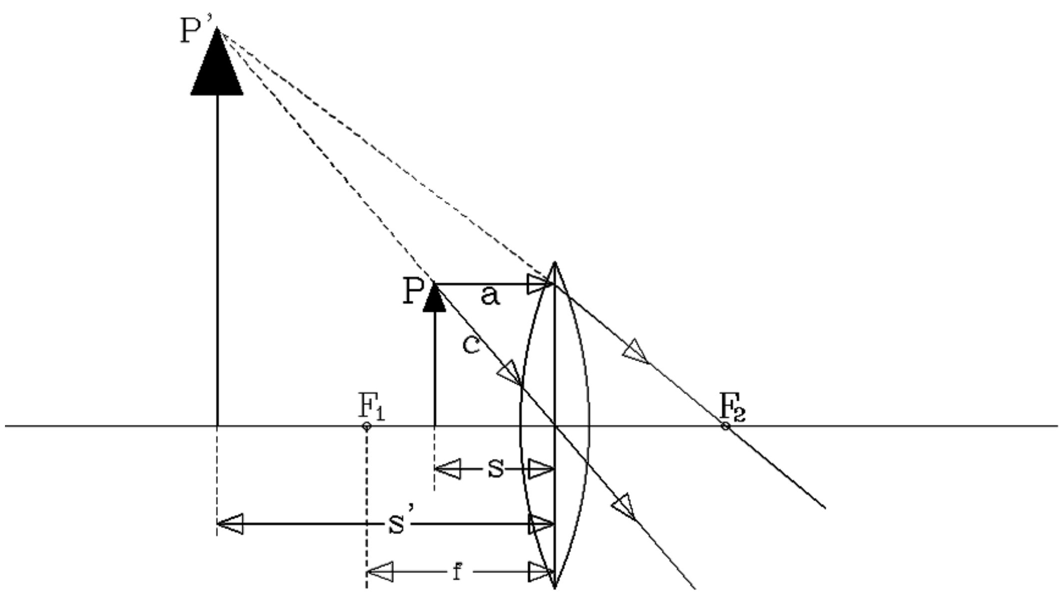
$$\frac{1}{f} = \frac{1}{S} + \frac{1}{S'}.$$

The sign convention that we have used in deriving this equation, sets *f* is positive for a converging lens, *S* is positive on the side of the lens where the object is placed, and *S'* is positive on the other side of the lens. For $S > f$, *S′* is also positive and a real image exists on the other side of the lens, as shown in the figure above. If the object were placed inside the focal length, i.e. if $S < f$, then the solution of the lens equation would give a negative value for *S′* and the image would appear on the same side of the lens as the object. In section 3.4, we will see that this negative solution for *S′* corresponds to a virtual image located on the left side of the lens.

The magnification of the image is the ratio of the image height to the object height. Eq. (1) shows that this magnification is numerically equal to S'/S, the ratio of image distance to object distance. The figure above also shows that the real image, formed when $S>f$, is inverted.

2.4 Virtual Images

Consider the following thought experiment, perhaps while looking at the figure in Section 2.3. Bring an object that is farther than a focal length from the lens closer and closer to the focal point. As you approach the focal point, the image recedes farther and farther away. When the object reaches the focal point, the rays exit parallel (and the image is infinitely far away). (Remember that this is how we define the focal point, and how we will determine the focal length in section 3.2.) As the object gets even closer to the lens, as shown in the figure above, the rays must diverge as they exit. If you extend the outgoing rays backward (behind the lens), you find that they intersect behind the original object. A virtual image is formed. It cannot be captured on a screen, but you can see it as you look through the lens - the object appears at the position of the virtual image and enlarged.

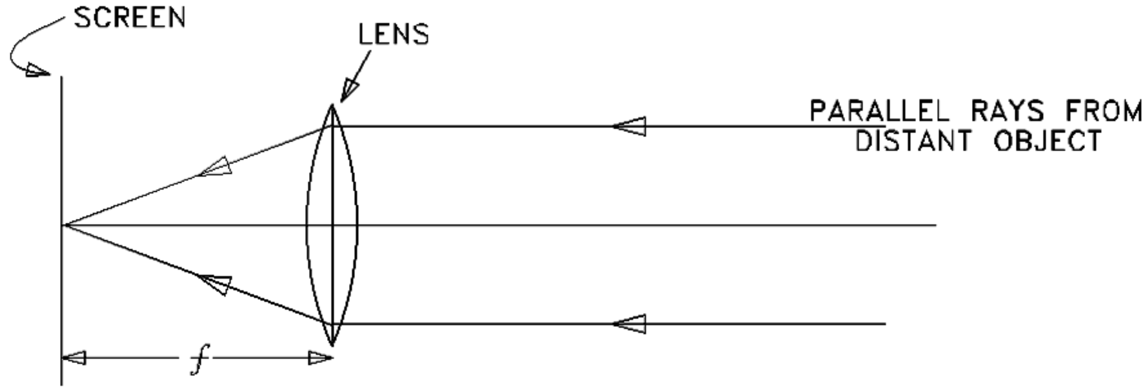


3. Experiments

Quantitative measurements using simple thin lenses will be used here to illustrate the basic features of geometrical optics. It would be deceptive, however, to stress precision in these measurements, since precise optical instruments utilize corrected, multi-element lenses. Instead, you will use a crude “optical bench” - a horizontal plastic meter stick, with lenses, screens, etc., mounted on riders that slide along the stick. The centers of all the components should be aligned at the same height so that measurements are performed primarily along the principal axes of the lenses. Note that there is a ridge on the aluminum clamps that fits directly into the groove on the meter stick. Do not tighten the screws on the clamps too much, and do not leave them so loose that things fall off and break.

Here we describe measurements of the focal length of a thin converging lens by two simple methods related directly to the definition of *f*. Time permitting, you should perform all parts of the experiment for a green lens (first) and a yellow lens.

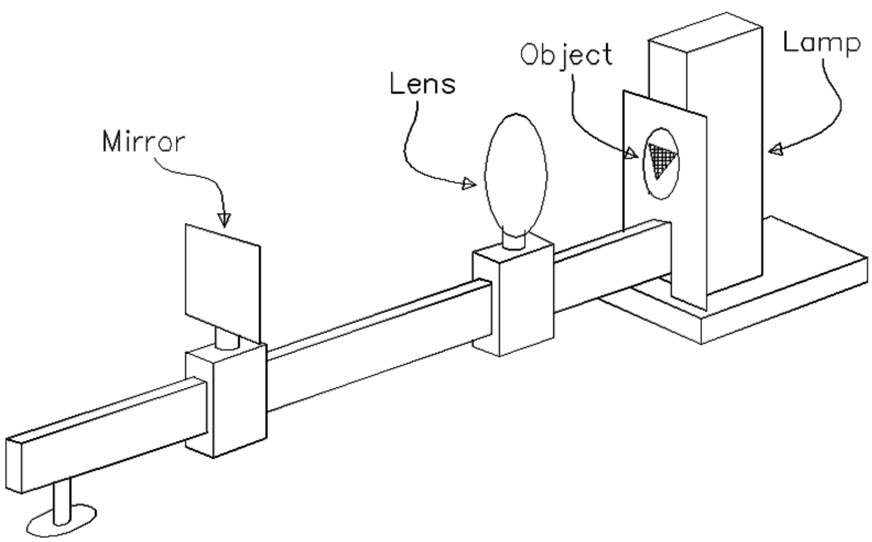
3.1 Measuring the Focal Length – I



Lower the lights in the room and open the window shade a little. Using a lens and a piece of paper, focus the image of an object outside the laboratory window (such as the façade of Teachers College) onto the paper. Based on the fact that light from a distant source may be safely assumed to contain only parallel rays, the distance between the lens and the paper should be the focal length of the lens. You may already suspect that this method is not very precise and that you will get large uncertainties. The method described in the next section will provide more precise results.

- Measure this distance and estimate uncertainty for both lenses.
- Take note of the orientation of the image on your paper.
- How could you improve this simple procedure (other than going to part 2)?

3.2 Measuring the Focal Length – II



Set up your optical bench as shown in the diagram above: place an object on one side of the lens and a mirror on the other side.

When the object is in the focal plane of the lens (when $S = f$) all the rays from a point on the object which pass through the lens will emerge in a parallel bundle (much like the figure shown in section 2.1). The rays remain parallel after reflection from the mirror. Any reflected parallel bundle will then pass back through the lens and converge at a point in the original focal plane.

The location of the focal plane can thus be determined by varying the object-to-lens distance until a sharp image is formed on the object screen itself, and then the object-to-lens distance equals the focal length. By using the backlit small triangular screened opening as an object, you can observe the image superimposed on the object and move the lens until the image is sharp (you may need to offset the image slightly to see this). It may be difficult to get a completely sharp image, but do your best to optimize the sharpness of the grid lines.

- Is the image erect or inverted? Is it magnified?
- Is the distance from the lens to mirror important? Explain why or why not.
- Measure the focal length, including a realistic uncertainty.
- Is the second result the same within uncertainty as in the first measurement (part 3.1)?
- Give the main sources of error.

3.3 Lens Equation

Use the same setup as in section 3.2 but instead of the mirror, use a screen. Place the lens more than a focal length away from the object and adjust the position of the screen until you get a sharp image on the screen. By measuring S and S', you can verify the lens equation.

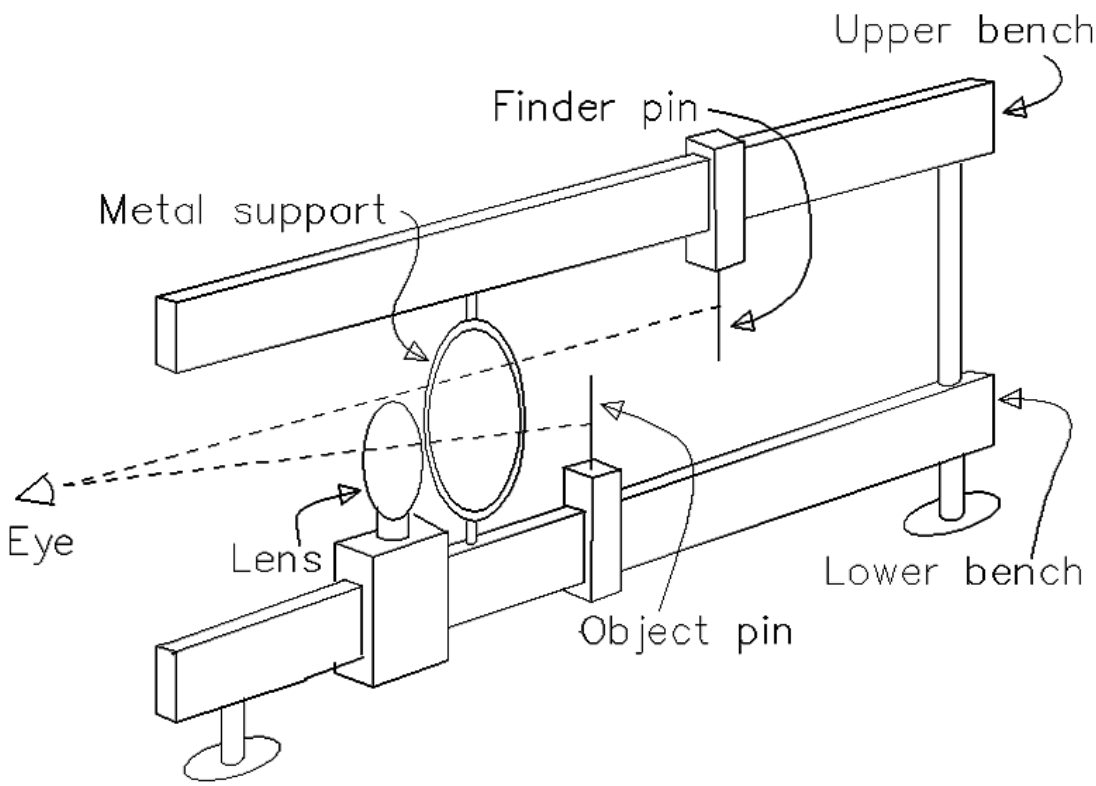
- Measure S and S` for five different pairs, including uncertainties. Plot a graph of $1/S$` vs. $1/S$, with the uncertainty represented as error bars. Draw a best-fit line and get the value of f from the intercepts with the axes.
- Is the value of f consistent with the value from the previous measurements? Is the value of the slope of the line what it is supposed to be?
- Comment on how well the line fits your data points.
- Which of the three methods used so far should give you the best estimate for f? Explain why.
- Give the main sources of error and how you could improve this experiment.

3.4 Virtual Images

So far, we have only dealt with cases where the object is located more than a focal length from the lens. What happens if we put the object closer than the focal length ($S < f$)? We know from section 2.4 that this causes a virtual image to be formed on the same side of the lens as the object.

How can you locate the virtual image experimentally? The rays do not actually converge, so you cannot capture the image on a screen, like you can for a real image, as shown in section 3.2. You can, however, use the parallax method, which was used to locate the virtual image of a pin formed by a plane mirror in the previous experiment (Experiment 1-4).

First, verify the procedure without the lens installed. Use a finder pin, which slides under another meter stick mounted above and parallel to the optical bench, as shown in the figure below. The parallax method can be tested by placing an upright object pin at a fixed point on the lower meter stick. While sighting along the bench, slide the finder pin until it no longer seems to shift with respect to the fixed pin when you move your head sideways. The finder pin should then be directly above the fixed pin.



Finally, place the same thin lens used the previous parts of the experiment in front of the object pin at a distance *S*, which should be about half of *f*. Locate the virtual image of the object pin by sighting along the bench as before and moving the finder pin to eliminate parallax between the finder pin (viewed above the lens) and the image of the object pin (viewed through the lens). This procedure may be easier (at least on your neck) if you look from about a meter away from the lens

Repeat this procedure for several object positions and read off *S* and *S`* (including uncertainty).

- Add the new 1/S and 1/S' points to your previous graph (from Section 3.3) and see whether the measured positions of the virtual image follow the prediction of the lens equation. Is your value of *f* consistent with the previous values?
- In your *1/S`* vs. *1/S* diagram, where should points corresponding to virtual images be located?
- Give the main sources of error for this part.

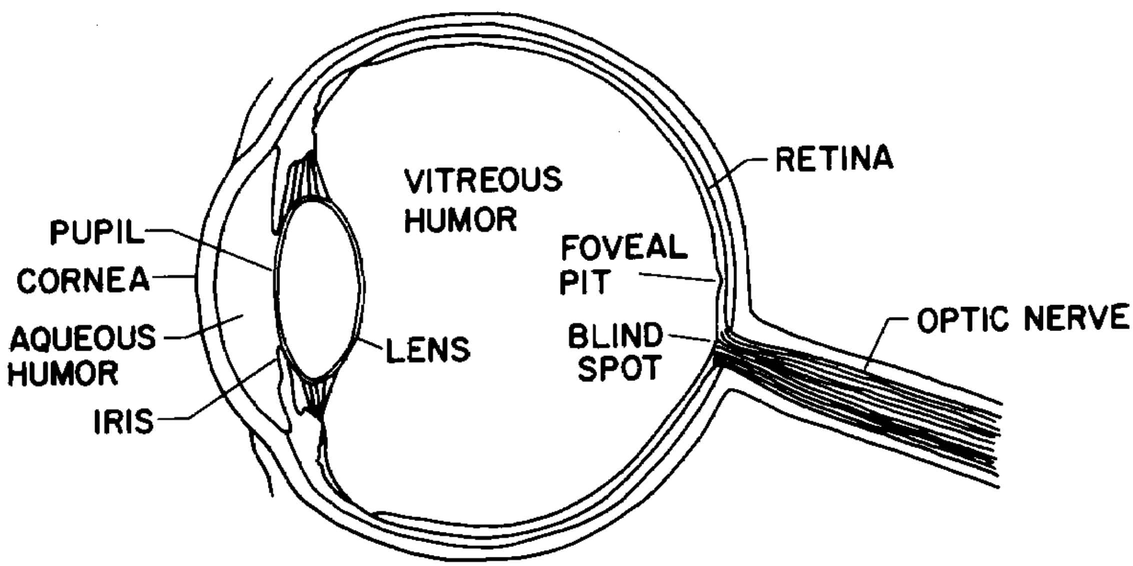
Please disassemble all parts of the experiment and leave everything the way you found it.

4. Applications (to everyday experience)

The most important application of lenses (to us) is the human eye! The retina is located a fixed distance from the lens. We need the ability to focus objects located at different distances in front of the eye onto the retina. These specifications require us to have an adjustable lens (with variable focal length). Adjusting the focal length is accomplished by deformation of the lens through contraction of the ciliary muscles. If the eye views a distant object, the muscles are relaxed and the lens is rather flat, with a long focal length. If the eye must examine a nearby object, the muscles contract and the lens becomes rounder, with a shorter focal length. With advancing age, the lens loses its flexibility so that the eye loses much of its ability to adapt to objects at close distances. (A common misperception is that this can be compensated by “eye exercises”, which would be the case if the problem were muscles. But the problem is not in lost vigor of the muscles, but in decreased flexibility of the lens!)

The two most common optical defects are nearsightedness and farsightedness. In a nearsighted (myopic) eye, the focal length is too short even when the ciliary muscles are completely relaxed. Thus, parallel rays from a distant object come to focus in front of the retina and fail to form a sharp image on the retina. Vision of distant objects is blurred. Eyeglasses with diverging lenses correct this condition.

In a farsighted (hyperopic) eye, the focal length is excessively long, even when the ciliary muscles are fully contracted. Hence, rays from a nearby object converge toward an image beyond the retina and fail to form a sharp image on the retina. Eyeglasses with converging lenses can correct this condition.



Reference: see Physics, by Ohanian, for further information.

Picture from: Daniel Malacara: Geometrical and Instrumental Optics.

5. Lab Preparation Examples

Lenses:

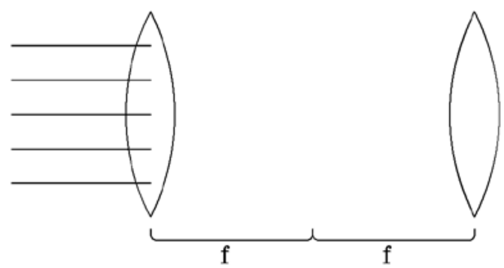
1. What kind of lens do you have if you find a lens with -50 written on it?

2. In section 2 of the experiment, you measure 25 cm between the mirror and the lens and 10 cm between the lens and the object/screen. What is the focal length of the lens?

3. You find a pair of glasses and you find out that can focus sunlight on a spot 5 cm away from the lens. What is the focal length of the lens?

Ray Diagrams:

4. Draw the ray diagram for the following lens-object system!



Lens Equation:

5. If the focal length of a lens is f = 100 mm. You place the object at a distance S = 250 mm. Where will you find the image? Is it a real or virtual image?

6. If the focal length of a lens is f = 100 mm. You place the object at a distance S = 50 mm. Where will you find the image? Is it a real or virtual image?

7. If the focal length of a lens is $f = 100 \pm 10$ mm. You place the object at a distance $S = 150 \pm 20$ mm. Where will you find the image? Is it a real or virtual image?

8. For what value of S is the image distance $S`$ equal to the object distance S, given the focal length f.

9. Draw a *1/S`* vs. *1/S* diagram using the values listed below. Make a best curve fit and determine *f*.

S in mm			*S`* in mm			*1/S*	*1/S`*
100	±	10	100	±	10	±	±
150	±	15	75	±	5	±	±
75	±	10	150	±	10	±	±

Virtual Image:

10. Given f = 100 mm, what is the position of the virtual image if S = 50 mm?

11. For S = 20 ± 5 mm and $S`$ = -40 ± 5 mm, what is the value of f?

Experiment 1-6

Geometrical Optics III: Optical Instruments

1. Introduction

In this experiment, we apply the ideas of geometrical optics to build a magnifying glass, microscope, and telescope and measure the magnifications of these systems.

2. Theory

2.1 The Near Point

In the human eye, light is focused on the retina by the elastic lens. Though the lens can change its focal length (through contractions of the ciliary muscles), the range of focus is limited. The closest point at which the eye can get a sharp image of an object is called the near point. The near point is different for every person, but the value of 25 cm is an average over a random segment of the population. We use this standard reference of 25 cm here. Young people with good eyesight may have a near point as close as 10-15 cm, and very young children have an even smaller one. You will determine the near point for your eyes in the course of this experiment.

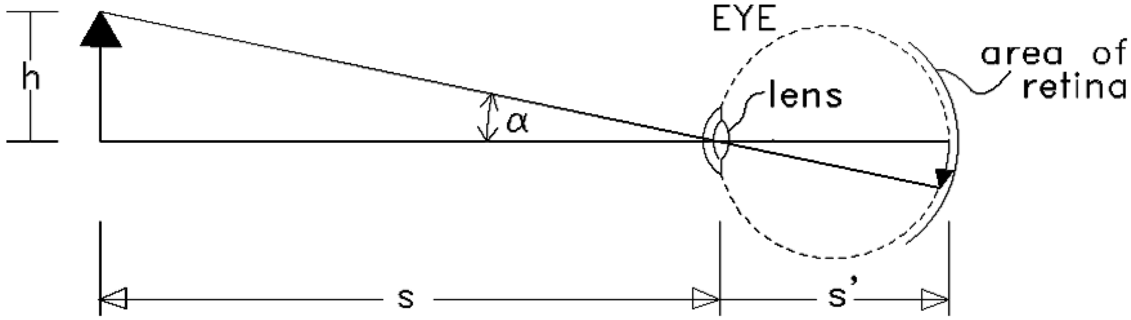
2.2 Resolution

In art museums, you probably observed the following effect. As you look at an impressionist painting from a distance, the picture appears sharp and easily identifiable. But as you approach the picture, you observe that it is made up of a number of smaller objects, large dots or blobs, and is quite coarse-grained.

What happened? Your retina consists of an array of light receptors, somewhat like the pixels in a video camera. When you are far enough away from the picture, all light rays from an object like a big dot fall onto a single receptor. Your visual impression from a long distance therefore consists of various sharp points. As you approach the picture, rays from this object begin to strike several receptors and you recognize that the blob is an extended object. If you look at the same picture, but shrunk by a factor of 10, you observe that you can get much closer before seeing the individual blobs. Such effects are related to the resolution limits of the eye.

The figure on the next page illustrates how an image appears on the retina. Note that the image is inverted on the retina. We actually “see” objects upside down. (Our brains do the work of inverting it back!) When we view an object, its size is determined by how big the object’s image is on our retina.

From the figure below, we see that the size of the retinal image is directly proportional to $\tan\alpha = h/s$. Clearly, there is a minimum value of α that can be resolved on the retina.



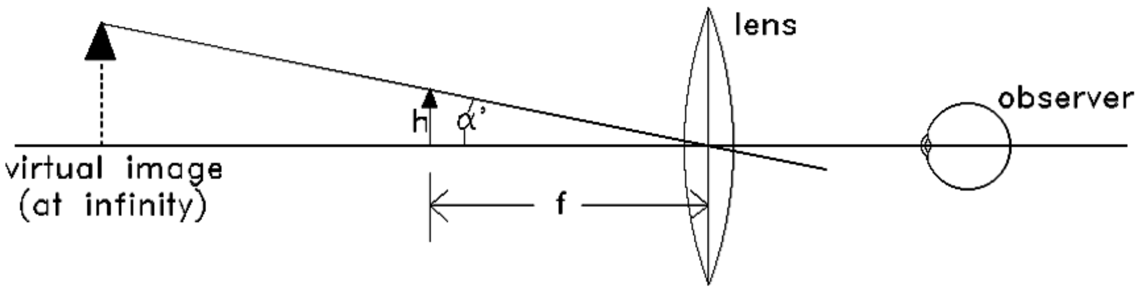
2.3 Need for Magnification

Suppose a person with a near point of 25 cm examines the object shown in the figure above. To see the most detail, the person should view the object so that it appears to be as large as possible while remaining in focus. As discussed above, the size of the retinal image varies as $\tan\alpha = h/s$. This means that the retinal image gets larger as s gets smaller (i.e., as the object is brought closer to the eye). The largest focused image this person can see is obtained when s = 25 cm. The size of the object when viewed at 25 cm is defined to have a magnification of unity. Is there any way to see more detail on the object, or, equivalently, can the object be made to appear larger? The answer is yes, and the simplest means of making the object appear larger is to use a magnifying glass.

2.4 The Magnifying Glass

A magnifying glass is a simple converging lens with short focal length that can make an object's image on the retina larger. How does it work? One limitation to the detail you see on an object is the limited angular resolution, which is due to the limited size of the individual receptors. (There is nothing we can do about this.) But the other limitation is that we cannot make the retinal image larger by bringing the object closer than the near point. There is a way to overcome this limitation, and it is used in all optical magnifying devices.

Consider the object and lens of focal length *f* shown in the figure below.



The object is placed a focal length *f* away from the lens. Using the lens equation, we find that a virtual image is formed at infinity as shown. (If you feel uncomfortable thinking of an image at infinity, you might prefer to think of it as simply a very large distance.)

If the rays of light from the ends of the object that pass through the center of the lens are extended to the left, they also pass through the ends of the image. (If this is not obvious, return to Experiment 1-5 and review the rules for ray tracing). This means that the angle $\alpha' = \tan^{-1} h/f$ is the same for both the object and the virtual image. Remember what an image represents - looking at the object through the lens can be represented as looking directly at the image. Since the image is very far away, its size does not appear to change as the eye (to the right of the lens) is moved small distances back and forth.

What actually happened when you put the converging lens one focal length in front of the object? The rays leaving the lens that actually hit the eye (not the ray shown in the figure) will be nearly parallel and therefore the rays appear to come from an infinitely far object – unless you are near-sighted, your eye can totally relax.

When we view the image, its size is proportional to $\tan\alpha' = h/f$. The <u>angular magnification</u> of the magnifying glass is defined as:

$$M = \frac{\tan\alpha'}{\tan\alpha} = \frac{h/f}{h/25\text{ cm}} = \frac{25\text{ cm}}{f\text{ cm}}$$

The magnification *M* represents the ratio of the <u>apparent</u> size of the object when viewed using the magnifying glass to that of the object when viewed directly at a distance of 25 cm.

If *f* is less than 25 cm, the lens increases the apparent size of the object and permits you to see more detail. Viewing an object with a magnifying glass of focal length 12.5 cm produces a retinal image twice as large as that formed when the object is viewed directly 25 cm away.

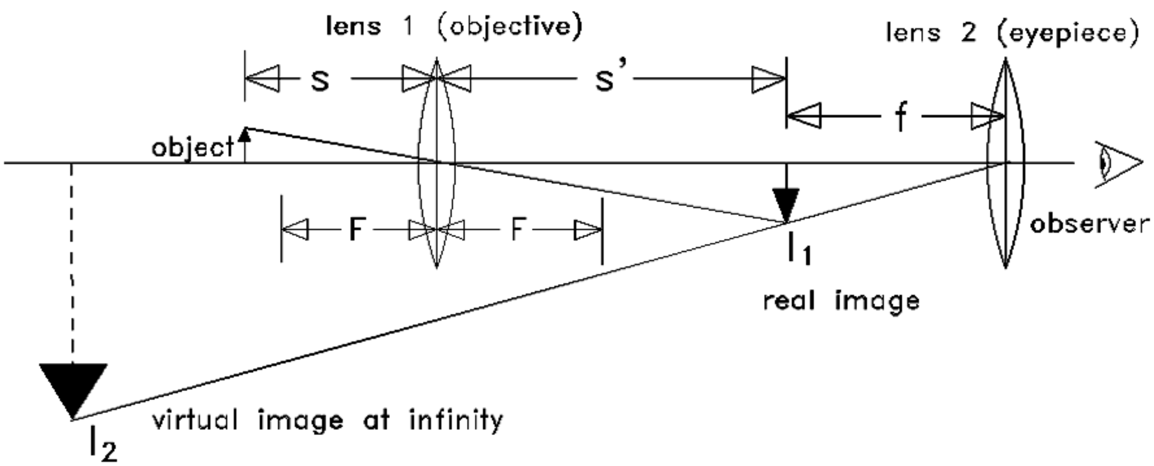
If a lens of focal length 25 cm is used as a magnifying glass, the apparent size of the object is the <u>same</u> as that seen when looking directly at the object 25 cm from the eye. Using the lens, however, permits your eye to relax because it views parallel rays, just as if the object were located very far away.

2.5 The Microscope

For various reasons, a single lens magnifier (such as a magnifying glass) can only provide good images with relatively small magnifications. To achieve higher magnification, a device known as a compound microscope can be used.

A compound microscope employs two lenses as shown in the next figure. The object to be viewed is placed just beyond the focal length, F, of the first lens, called the objective lens. The light collected by the objective lens forms a real (enlarged) image of the object at I_1. (The image is real because a screen placed at I_1 would display an actual inverted image of the object.) The second lens, the eyepiece, has a focal length f and acts as a magnifying glass that is used to examine the image I_1.

As discussed in the preceding section, the observer looking through the eyepiece (magnifier) sees a virtual image I_2 (located near infinity) of the real image I_1, acting as an object for lens 2.



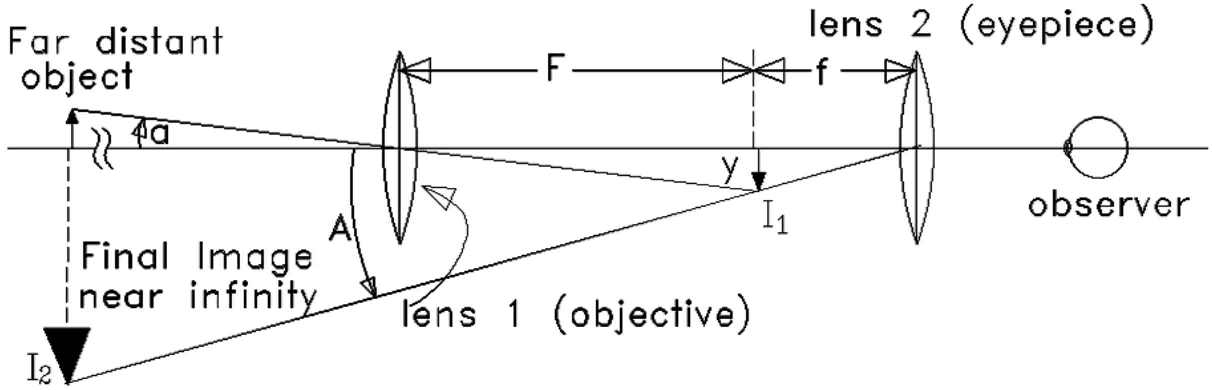
How big does the original object appear? Since the eyepiece acts like a magnifying glass, the image I_1 appears to be 25 cm/f times bigger than if viewed from the standard distance of 25 cm. The image I_1 is itself bigger than the object by a factor (as described in the previous experiment) of s'/s.

The total magnification of the microscope is then:

$$M_{\text{microscope}} = \left(\frac{s'}{s} \times \frac{25\,\text{cm}}{f} \right).$$

2.6 The Telescope (Optional)

The optical system of a refracting telescope has the same elements as those of a compound microscope. In both instruments, the real image formed by an objective lens is viewed through an eyepiece. However, the object of a telescope is distant, the incoming rays are parallel and the objective lens forms the first image I_1, near its focal point F, as shown in the figure below. (By contrast, for the microscope $s' >> F$.)



I_1 again serves as the object for the eyepiece. In order for I_2 – the final virtual image made by the eyepiece – to be near infinity (for relaxed viewing), the eyepiece needs to be located so that I_1 is also near its focal point f. Therefore, the distance between the objective lens and the eyepiece – the length of the telescope – is the sum of the focal lengths $F+f$.

The angular magnification of the telescope is easily derived by noting that a, the very small angle that the distant object would subtend for the unaided eye, is essentially the same as the angle subtended by the image I_1 from the objective lens. Then from the figure above we have:

$$\tan a = \frac{y}{F}$$

and angle A, subtended by I_1 at the eyepiece, we have:

$$\tan A = \frac{y}{f}$$

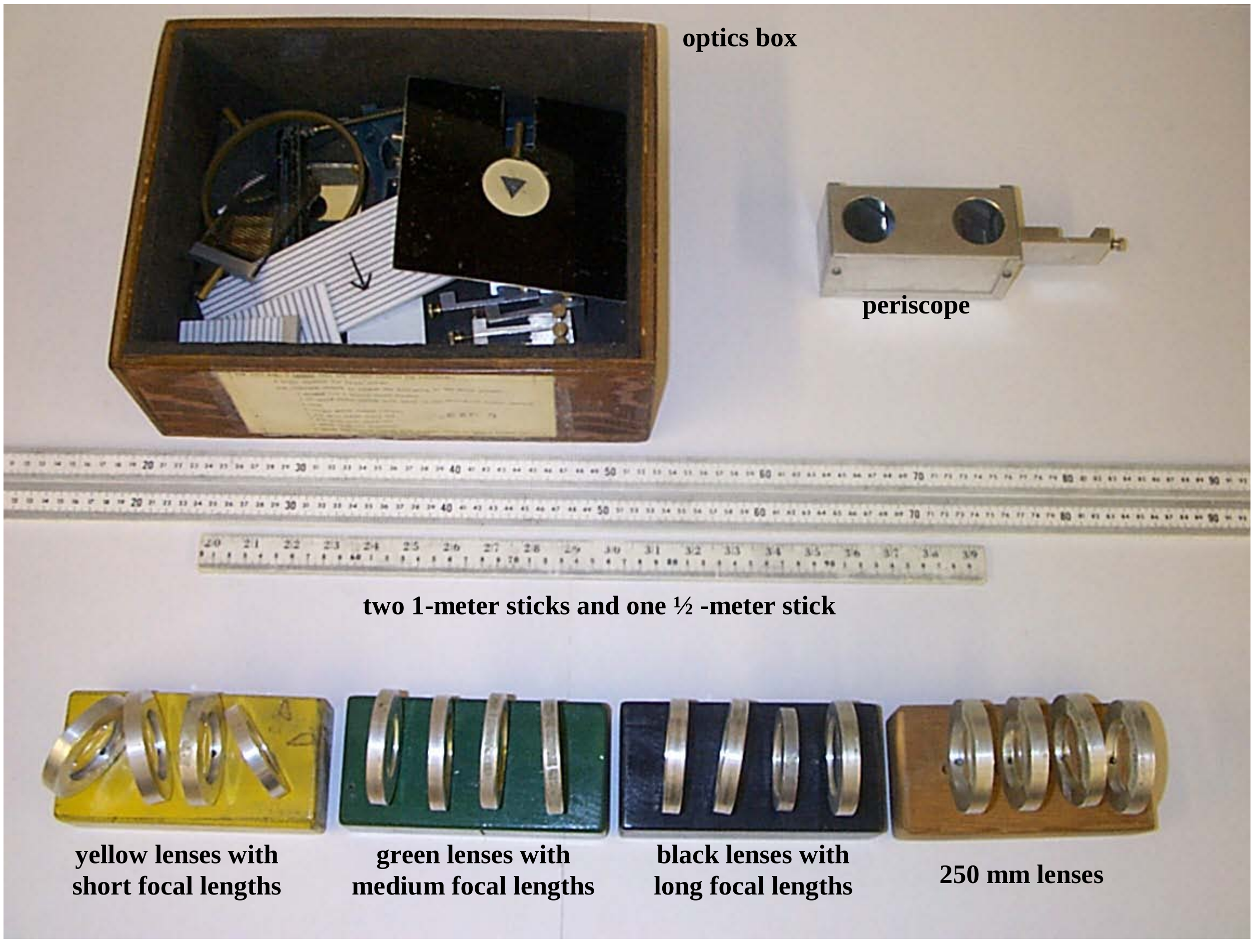
so:

$$M_{\text{telescope}} = \frac{\tan A}{\tan a} = \frac{y/f}{y/F}$$

$$M_{\text{telescope}} = \frac{F}{f}$$

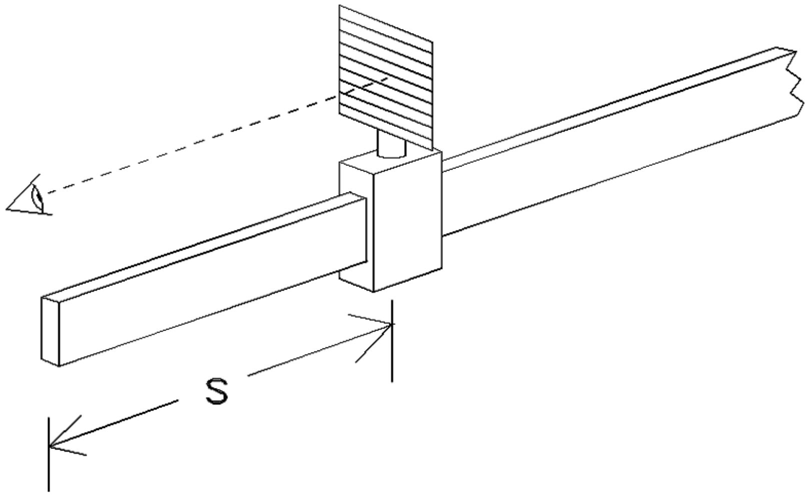
3. Experiments

Equipment List



Note: For the lenses you use (yellow, green) you first need to determine their focal length. You can do that using the same method as in part one of the previous lab (Lab 1-5: Geometrical Optics II).

3.1 The Near point



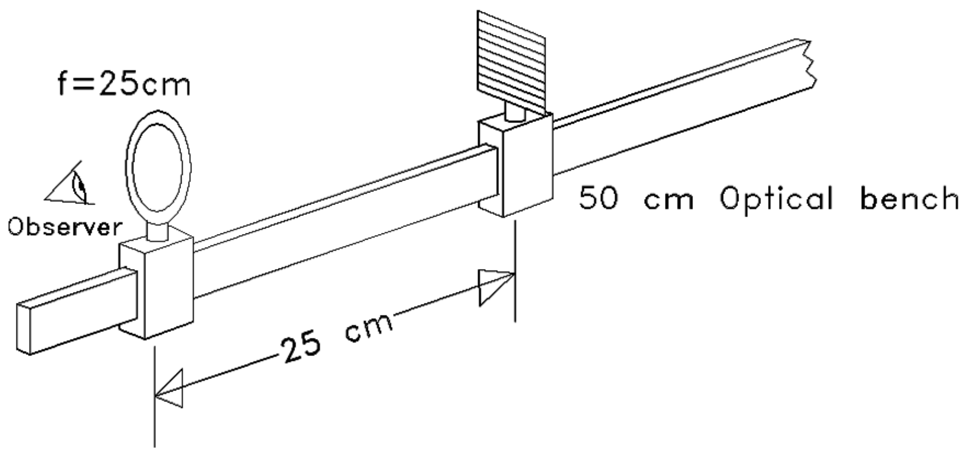
In the first part of the lab, you will measure the near point of each eye. Place one of the white ruled screens on an optical bench rider. Place your head so that one eye is at the end of the meter-stick optical bench. Move the screen as close to your eye as possible, consistent with the requirement that the screen remains in focus. The distance *s* is the near point for that eye. Repeat for the other eye.

- Is the near point the same for both eyes?
- How do these values for your eyes compare to the standard value of 25 cm?

If the values you obtain are different from the conventional 25 cm value, it should not be surprising; as you compare your values other people in the class, you will probably see a wide variation. So as to make quantitative comparisons of instruments, we use a standardized reference near point of 25 cm.

3.2 The Reference

In the next two sections, we measure the angular magnification of a magnifying glass and a compound microscope. These measurements are performed by comparing a ruled screen viewed of unity magnification with another ruled screen viewed through the optical instruments (magnifying glass and compound microscope) described above.



To form a virtual image of the ruled screen at infinity with unit magnification, we use a 25-cm-focal-length-lens "magnifying" glass. Place the 25 cm lens (no color) near the end of the 50 cm-long optical bench. Place the front surface of the ruled white screen exactly 25 cm away from the center of the lens.

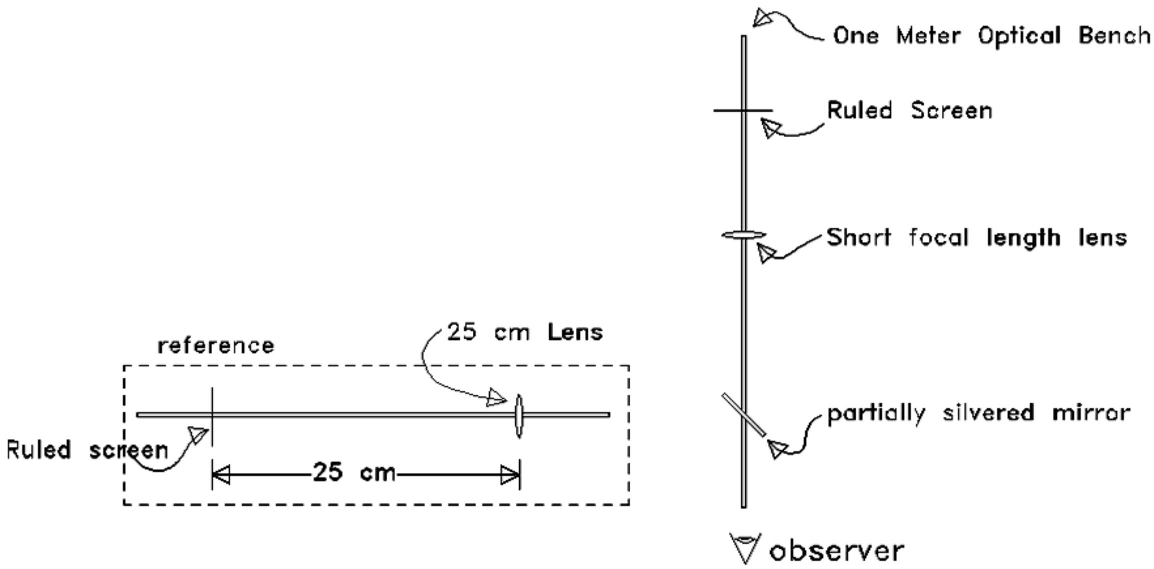
Look at the screen through the lens. Unless you are a little near-sighted (i.e. cannot clearly focus objects at infinity) the screen should be in sharp focus. Its apparent size should not change as you move your head back and forth along your line of sight (because you are actually viewing an image very far away and the motion of your head represents a negligible fraction of that distance).

The size of the object should appear exactly the same as if you looked at the object directly from 25 cm away. Why not just look at the screen directly and forget about the lens? Because then the screen would need to be exactly 25 cm from your eye. With the 25 cm lens, your eye can be anywhere to the left of the lens (in the figure) and the ruling on the screen should appear to be the same size.

This system serves as the reference for subsequent measurements of magnification made for other optical devices.

3.3 The Magnifying Glass

In this part, we set up a magnifying glass and measure the magnification. Given the formula for the magnification from section 2.5, we can compare the measurement with the prediction. The apparatus is shown in the figure below.



Leave the 25 cm reference as set up in the previous part. Mount a second white ruled screen and a short focal length lens (yellow) on the one meter optical bench, as shown in the figure above. Leave space at the end of the bench for a partially reflecting mirror, which will be located as close as possible to the short focal length lens.

Move the screen until the image is in focus. Look only through the centers of the lenses to avoid edge distortion. The rulings on the screen should appear enlarged.

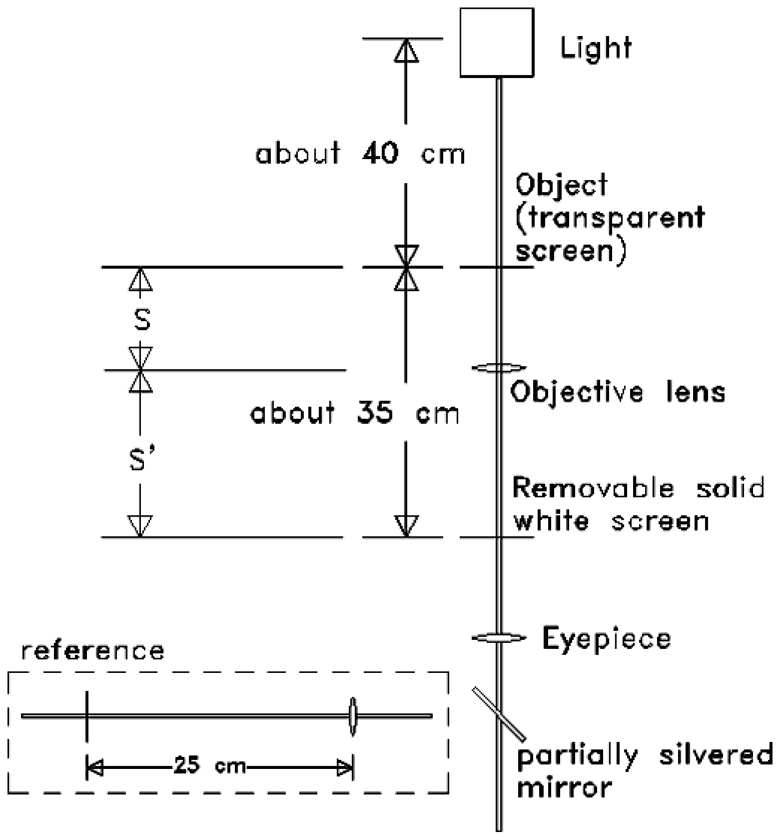
Insert the partially silvered mirror at a 45° angle as shown in the figure. Check that you are still able to see the ruled screen. If you have difficulty seeing the screen, illuminate it with the lamp supplied. Now position the 50 cm bench, on which you set up the reference image, as shown in the figure. By rotating the partially silvered mirror, and/or adjusting the position of the 50 cm bench, you should be able to see both the magnified and reference images.

- Count the number of divisions on the reference image that fall within one division on the magnified image. This provides the magnification, since the rulings on both white screens have identical spacing. Does your measurement of the magnification agree with the prediction?

The magnification you measure here should not depend on where you position your eye. If the magnification appears to change as you move your head along your line of sight, you probably do not have the magnified screen exactly a focal length away from the short focal length lens.

3.4 The Microscope

The figure below shows the apparatus for measurement of magnification of the microscope.



NOTE: you may find that the distance (S+S') should be more like 50 cm than the 35 cm noted in the diagram above.

Place the lamp at one end of the one-meter optical bench. (The ruler fits into a hole in the base of the lamp.) Use the transparent ruled screen as an object and place it in front of the lamp. If you have problems illuminating the transparent screen, you can put it closer to the lamp.

Position the solid white screen approximately 50 cm (not 35 cm as noted in the above figure) from the object.

Place a medium focal length lens (green), to be used as the objective lens, between the object and the screen. Position it so that the rulings of the object are focused onto the solid white screen. The real image of the rulings on the white screen will appear enlarged.

Now place a short focal length lens (yellow), to be used as the eyepiece, on the other side of the solid white screen. Position the lens as a magnifying glass to view the back of the white screen. (You should be able to see irregularities in the screen's surface - do not mark it.)

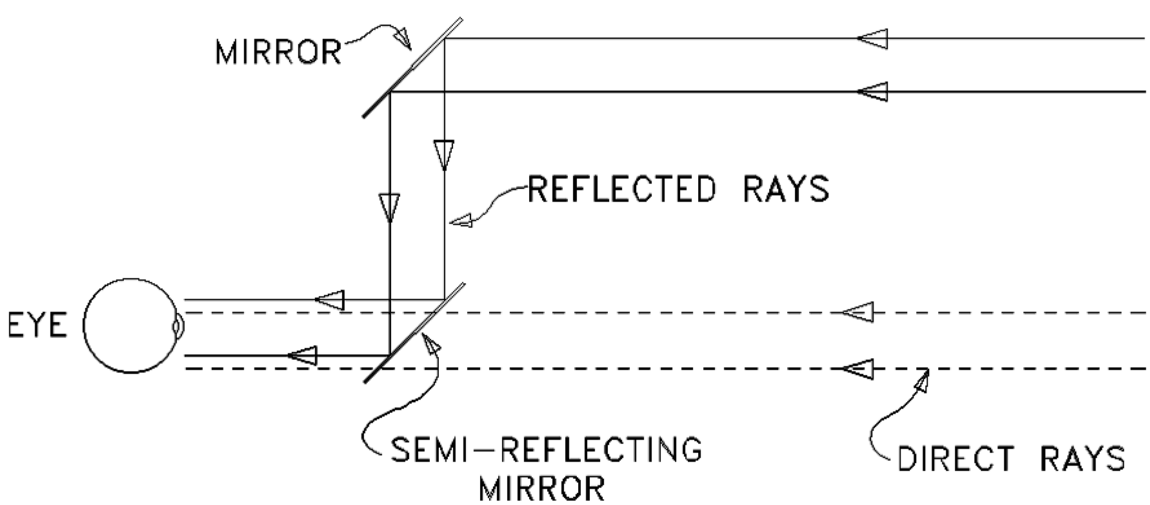
Remove the white screen. Looking through the eyepiece lens, you should see a highly enlarged view of the object. Because of the distribution of light passing through the microscope, you must place your eye a certain distance away from the eyepiece in order to see the entire image.

Use the reference image to measure the magnification of the microscope. If you have trouble getting proper illumination on both the reference and magnified images, try covering one side of the transparent screen (object) with paper. This will leave one side dark and the reference image should be visible. Note that the rulings on the transparent screen have one-half the spacing found on the white ruled screens. Therefore, if you measure ten reference divisions to one magnified division, the magnification of the microscope is twenty (not ten).

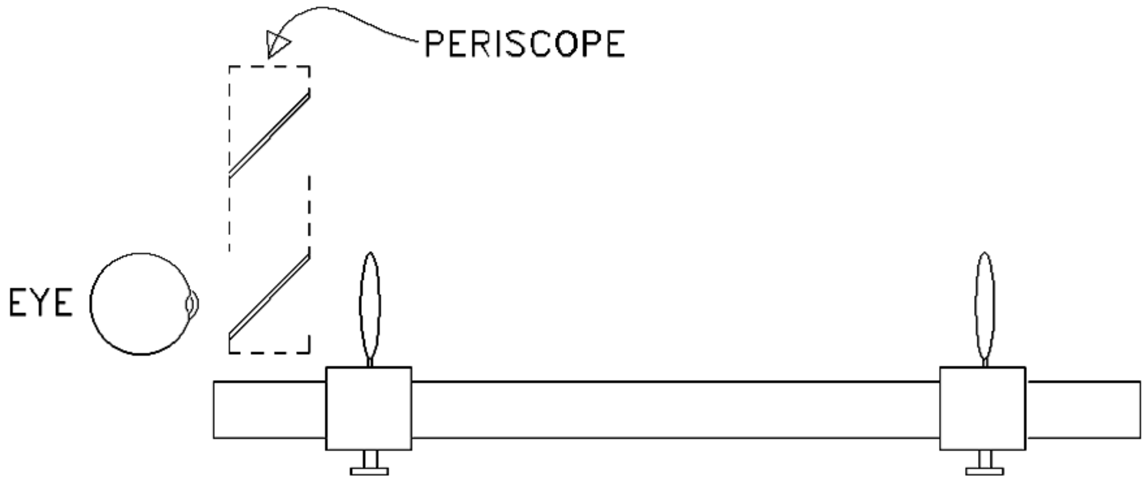
- How does the measured magnification compare with the prediction?
- Draw a careful ray diagram of this instrument (using graphical definitions of "thin lens" and "focal point" and following three rays from a point on each object to a corresponding point on each image). Indicate whether each image is real or virtual, inverted or erect.
- Note the main sources of error in the experiments.

3.5 Optional: The Telescope

The last part of the experiment is optional. You can try it if you have time left.



We assemble a telescope and measure its magnification. Since we want to look at a distant object, we cannot use the 25 cm reference. Instead, we use a periscope so that we can simultaneously look directly at an object and through the telescope. Comparison of these permits a quantitative estimate of the magnification.



Set up a telescope by using a long focal length lens (black) for the objective and sighting on a distant object outside the laboratory window. If the distant object is sufficiently bright, you may be able to locate the intermediate image on a translucent screen and proceed as you did for the microscope. If not, set the two lenses in place directly, as indicated in section 2.6.

With your eye about 5 or 10 cm behind the eyepiece, focus the telescope on a distant object. Place the periscope between your eye and the telescope, and superimpose the reflected (unmagnified) image onto the magnified one.

- Measure the approximate magnification of the telescope by comparing the sizes of the two images. (Large bricks on the side of a building make a good object to use for the comparison.)
- Does the measured magnification agree approximately with that derived from the focal lengths of the two lenses?
- Note that this telescope gives an inverted virtual image. Can you make a telescope that gives an erect image and/or one that you can photograph?
- Draw a careful ray diagram for the telescope just as you did for the compound microscope, and indicate the type of each image.

4. Applications

It may seem strange that we provide yet another example based on the human eye, since you already had that in the last experiment. As with many real-life applications, you are only told part of the story at first (with the complexities left to later). The eye should better be described as a system consisting of two lenses.

The index of refraction of the fluid within the eye is about 1.34 (nearly the same as water). The index of refraction of the solid lens is about 1.44. The difference in the indices of refraction is not large, so the refraction at the interface between them is not strong. Therefore, though the lens provides adjustments in focal length necessary to form the image on the retina at different object distances, the main contribution of focusing the incoming light is refraction at the cornea. This tissue layer separates the eye fluid (or humor) from the air. The difference in indices of refraction between air (1.00) and the aqueous humor (1.34) is much larger and creates substantial refraction. This means that the eye is more precisely treated as a system of two lenses, in which the second lens is variable so as to fine tune the focal length of the system.

Several different optical malfunctions (e.g. farsightedness/nearsightedness) were described in the application part of the previous experiment. These can also arise from malformations of the eyeball. As described before, using an additional lens either in the form of glasses or contact lenses can compensate these.

But a more recent approach is to change the focal length of the first lens (cornea) directly. A high intensity laser beam can be used to evaporate thin layers of the cornea and thus provide the cornea with a new shape. If the curvature of the cornea is increased, the light is focused more (corrects farsightedness) or if you decrease the curvature, the light is focused less (corrects nearsightedness).

5. Lab Preparation Problems

Magnification and Magnifying Glass:

1. The angle subtended by an image with an optical device is $\alpha' = 10^o$ and without it, the object subtends $\alpha = 7^o$. What is the magnification?

2. What is the magnification of a magnifying glass with a focal length of 10 cm?

3. Assume your eyes have a focal length of 25 cm. What is the advantage for your eyes if you still use a magnifying glass of focal length 25 cm to read a book instead of holding the book at a distance of 25 cm?

Microscope:

4. Assume you have a microscope and measure s' = 30 cm and s = 10 cm. Your second lens has a focal length of 15 cm. What is the magnification?

5. Assume your microscope has a focal length of F = 10 cm for the first lens and f = 10 cm for the second lens. What is the magnification if you choose s = 15 cm? What if you choose s = 12 cm?

6. You build a microscope with a first lens of F = 5 cm and a second lens with f = 10 cm. You separate these two lenses by a distance of 25 cm. What will the magnification of your microscope be?

   Hint: You first have to calculate s' and then s!

Telescope:

7. You build a telescope with two lenses. One has a focal length of 5 cm, the other has a focal length of 2 m. In what order do you want to place these two lenses. Which should be the objective lens, which the eyepiece?

8. For the telescope described in 7, what is the magnification?

9. You want to build a telescope that can magnify far away objects in the following way. Everything that appears to be separated at an angle of 1' (1/60 of a degree) when viewed without the telescope will appear to be separated by 1^o when viewed through the telescope. You have a 5 cm lens for the eyepiece. How do you have to choose the objective lens such that the magnification is as described? How long will your telescope then be? (The length of your telescope is the separation of the two lenses.)

Experiment 1-7

Conservation of Momentum

1. Introduction

Momentum is one of the central quantities in physics and, like energy, is bound by a conservation law. Although conservation of momentum is less intuitive to many people than conservation of energy, conservation of momentum can be applicable in situations where conservation of energy is not. Sometimes, conservation of energy may not be easily utilized since it can be difficult to keep track of all possible energy contributions (such as energy contribution that deforms the metal of two cars as they collide), but conservation of momentum may still be helpful. For some problems, momentum conservation is essential. We will see that specific problems, like the ballistic pendulum that you will study today, require both energy and momentum conservation for their solution. Others, like the air trough, permit prediction of details of motion in collisions where mechanical energy is conserved (elastic) as well as where mechanical energy is not conserved (inelastic). This lab is designed to provide you with an intuitive, yet quantitative, sense of momentum and some of its important applications[1].

Remark: You must prepare some derivations (for the pendulum, the velocity ratios of the riders, and the elastic collision) at home; otherwise you may not finish the lab on time!

2. Theory

2.1 Momentum Conservation

Momentum is a vector quantity, so it has magnitude and direction. To add or subtract momenta, use the usual rules of vector addition. In this lab, we deal only with momentum in one dimension, so the vector property is applicable only in the sense that if two objects move in opposite directions, their momenta have opposite signs. This is the source of most mistakes when performing calculations with momenta, so be careful.

Momentum is the product of mass and velocity:

$$\boldsymbol{p} = m \cdot \boldsymbol{v}.$$

A fundamental property of nature is that the total momentum components of any closed system[2] are conserved in any physical process.

[1] See Chapter 9 of Fundamentals of Physics, 8th Ed., by Halliday, Resnick & Walker.
[2] A closed system is one in which the sums of force components external to the specified system are negligible.

2.2 The Air Trough

Using the riders in the air trough, we will test momentum conservation by measuring total momentum before and after collisions. Since momentum is the product of mass and velocity, we should, in principle, measure the mass and velocity of each object before and after the collision. We will measure the masses with a balance. But rather than measure the velocities directly, we will use a trick that simplifies the measurements.

We position the riders in such a way that, after they collide and move in opposite directions, they touch the ends at the same time. Since the riders started at a common position (which we can record), we can determine the distance they traveled to the two ends of the trough (called s_1 and s_2). Since the times are equal, the ratio of the velocities is equal to the ratio of the distances:

$$v_1/v_2 = (s_1/t)/(s_2/t) = s_1/s_2.$$

This comes from simply using $s=v\cdot t$.

In order to predict the velocity ratio of the two riders we are going to use momentum and energy conservation.

A). In the first experiment performed on the air track, a large rider (of mass *M*) begins at rest, while a small rider (of mass *m*) with an initial velocity collides with it elastically. Therefore, we can use both momentum and energy conservation to determine the expected final velocity ratio.

Momentum conservation: $m\cdot v_{1i} = m\cdot v_{1f}+M\cdot v_{2f}$

Energy conservation: $\frac{1}{2}\, m\cdot v_{1i}^2 = \frac{1}{2}\, m\cdot v_{1f}^2+\frac{1}{2}\, M\cdot v_{2f}^2$

After some algebraic manipulation, the equations can be combined to read:

$$v_{1f}/v_{2f} = \tfrac{1}{2}(1-M/m)$$

B). In the second experiment, both riders begin at rest and are exploded apart using a Tesla coil. This is considered like the reverse of an inelastic collision in which momentum is conserved but not energy.

Momentum conservation: $0= m\cdot v_{1f}+M\cdot v_{2f}$

2.3 The Ballistic Pendulum

A ballistic pendulum is an instrument used to indirectly measure the velocity of a projectile. In this experiment, a small metal ball strikes a stationary pendulum (a can filled with clay) and sticks to it. The initial velocity of the ball can be deduced by observing the pendulum as it swings after the collision. In this experiment, we measure the velocity of the projectile and deduce how far the pendulum should move. This is then compared to the actual measured pendulum displacement.

It is easiest to analyze the problem by splitting it into two parts. In the first part, consider the projectile and pendulum in the time right before and right after the collision. The pendulum is initially at rest at its equilibrium point, whereas the projectile travels with an initial velocity and therefore kinetic energy. When the projectile strikes the pendulum, they stick together and move at the same velocity. Since an unknown part of the initial kinetic energy is used to deform the clay so the projectile can lodge into it, we cannot directly calculate the pendulum's subsequent motion from conservation of mechanical energy. However, momentum conservation will help us, since it is not affected by the deformation of the clay.

More quantitatively, consider the projectile to have a mass m and initial velocity v, and the pendulum to have a mass M and initial velocity of 0. The total initial momentum is $m \cdot v$. When the projectile and the pendulum stick together, they have total mass $m + M$ and we define their combined velocity to be V. The relationship from conservation of momentum is:

$$m \cdot v = (m + M) \cdot V$$

For the second part of the problem, we consider the motion of the pendulum after the collision. After the collision, we cannot use conservation of momentum since there are forces on the pendulum (gravity and the strings) from outside the pendulum which change momentum. Specifically, we measure the maximum height that the pendulum swings, h. Here, we can use energy conservation, because there are no uncontrollable energy losses (like the energy lost when the bullet deforms the clay in the first part of the problem). Conservation of energy requires:

$$\textit{Kinetic energy after the collision} = \textit{Potential energy at the top of the swing},$$

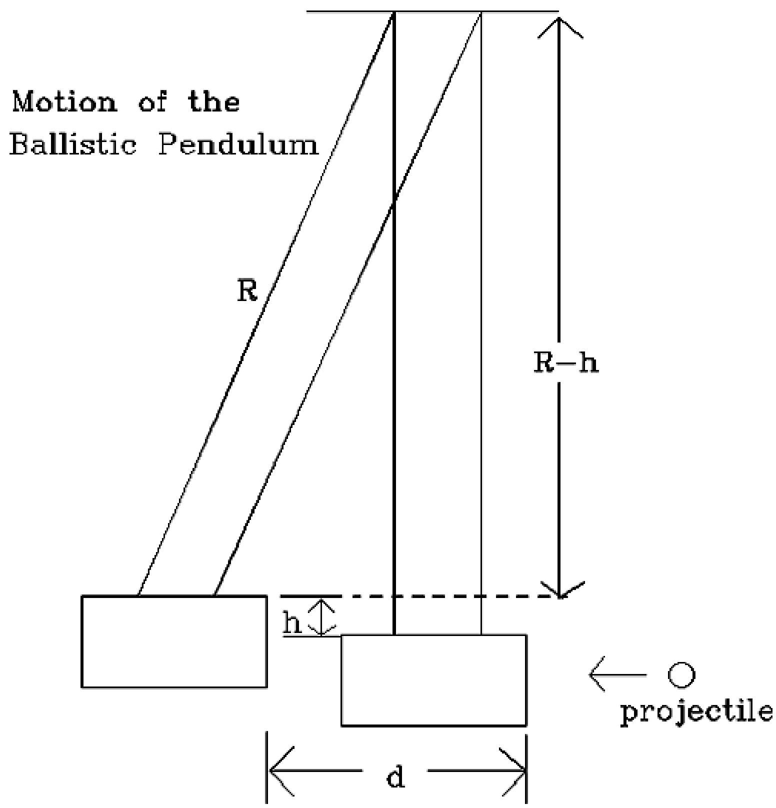
$$\tfrac{1}{2}(m + M) \cdot V^2 = (m + M) \cdot g \cdot h.$$

This relation permits us to calculate the velocity (just after collision) of the pendulum from the maximum height that the pendulum reaches. Combined with the earlier expression, we can use it to obtain the initial velocity of the projectile.

But it is difficult to directly measure the vertical rise, h. Instead it is much easier to measure the horizontal distance, d, that the pendulum swings through (see figure.) These quantities are geometrically related, by the Pythagorean Theorem, as:

$$R^2 = (R-h)^2 + d^2.$$

These three equations permit you to derive the dependence of *v* on the quantities *m*, *M*, *g*, *R*, *d*. Conversely, you can find *d* in terms of *m*, *M*, *g*, *R*, and *v*.

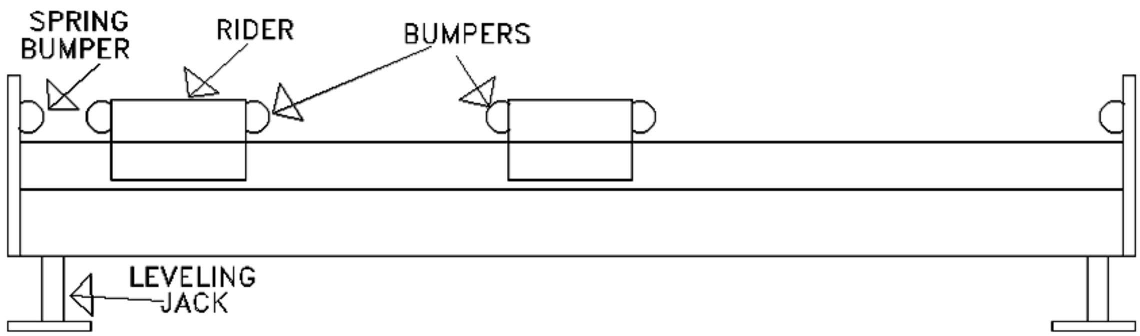


<u>Remark</u>: Make sure that you go through the derivation of the steps needed to arrive at the expressions for *v* and *d* before coming to the lab, since you will need them, and your TA is not going to derive them for you! Since the full expressions are rather cumbersome, you may choose to work in steps, calculating the error at each stage, before inserting values into the next step of the calculation.

3. Experiments

3.1 The Air Trough

In the air trough, we observe both elastic and inelastic collisions. The air trough provides a steady airflow so that the riders move along a layer of air and the effects of friction are negligible. Flexible steel bumpers, mounted on the riders and at the ends of the trough, provide almost perfectly elastic collisions.



Please handle the riders with care! Don't put them on the trough without air flowing and store them only on the felt covered holders provided. Make sure that the bumpers are inserted on both ends of the riders, and don't make violent collisions! (this also compromises your data).

Place a rider at rest near the center of the air trough. The trough may need a minor adjustment, using the leveling jack, to make it as close to horizontal as possible (note that there may be some bowing or sagging of the trough due to its weight).

The following two experiments are performed with the air trough.

A) <u>Elastic collision between two unequal riders</u>. Select a large and a small rider, measure their masses to determine the expected velocity ratio. Place the large rider at rest on the air trough and send the small one moving toward it with an arbitrary velocity. Adjust the positioning until you have found an initial position of the large rider such that the two riders hit the ends of the trough at exactly the same time (you will probably have to try this several times!). When you have determined this point, measure the distances that each of the riders traveled to reach the end (s_1 and s_2), and use that to determine the ratio of their velocities. Include an uncertainty estimate.

- Compare the experimental velocity ratio to a theoretical prediction from the masses of the riders
- What would happen if the two riders had exactly equal mass?
- What would happen if you placed the small glider stationary and send the large one on it? Why do we not want this to happen?

B) Inelastic separation of gliders. Connect the same two gliders together using the piston and the cylinder, and place a toy cap between them. Explode the cap using an electric spark from a Tesla coil. (This may require several attempts: you may need to clean the piston and cylinder, and be very careful as you place the toy cap.) Again choose the initial location so that the riders hit the ends at exactly the same time and determine the velocity ratio. Include an uncertainty estimate.

- Compare the experimental velocity ratio to a theoretical prediction from the masses of the riders

- After bouncing off the ends, do the two riders meet again at the position from which they started? Explain why or why not!

Safety remark: Handle the Tesla coil carefully. It produces several thousand volts and you or others can get a serious shock!

3.2 The Ballistic Pendulum

Safety remark: Do not have the gun loaded while you are taking measurements or setting up

In the ballistic pendulum experiment, you will shoot a small metal ball out of a piece of tubing (gun) and embed it in the "bob" of a pendulum. The pendulum consists of an open can filled with clay, and suspended by four strings. The velocity of the projectile is determined by measuring its flight time between a pair of photocells. Using the derived equations this permits you to predict how far (d) the pendulum should swing. You measure d by observing how far the pendulum pushes a small glider. The predicted and measured values can then be directly compared.

Measure all of the components of the ballistic pendulum that you need for calculations. (You may use the values for some of the pendulum parameters that are written next to the apparatus, but be sure to measure the mass of the pendulum itself using the scales.) Use the photogate timer to measure the time it takes the projectile to travel between the two photogates. The small rider on the ruler is used to measure the distance that the pendulum has traveled – you note the initial position and shoot the projectile twice without moving the rider in between.

- Calculate the velocity of the projectile from the light gate data. With this, calculate how far you predict the pendulum to have swung back. Compare this with the distance that the rider actually traveled according to your measurements

- The reason we shoot twice is that there is friction between the rider and the ruler on which it rests. By performing the experiment twice, we reduce the effect of this friction. (Can you explain why?)

- Describe the main sources of error in this experiment

- Are your prediction and measurement equal within a reasonable error?

Remark: If the clay is too stiff to perform the experiment properly and the projectile doesn’t stick to it, remove the can and put it under the lamp provided to heat it up so that the clay becomes malleable again.

4. Lab Preparation Examples

Propagation of Uncertainty:

1. Given $a = 1.5 \pm 0.5$ m and $b = 3.0 \pm 0.6$ m what is a/b?

Ballistic Pendulum:

2. You measure the following values for the pendulum experiment:

s	*t*	*R*	*m*	*M*
60 cm	0.01 s	1.5 m	10 g	1 kg

What is your predicted value for Δd?

Conversion m/s, km/h and miles/h:

4. What is 1 m/s in km/h?

5. How many miles/h is the speed of 200 km/s?

Air Trough:

6. In the elastic collision, the riders weigh m = 100 g and M = 250 g. What is the expected value for $v_1/v_2 = s_1/s_2$?

7. In the inelastic collision, the riders weight m = 100 g and M = 250 g. What is the theoretical value for $v_1/v_2 = s_1/s_2$?

Explanations:

8. You are sitting in a car of mass M. Another car with mass m = 500 kg crashes into you with a relative velocity of 10 m/s. Explain in a few sentences (and maybe equations) why you feel less impact if your car has M = 2000 kg as if your car has only M = 500 kg?

Experiment 1-8

Projectile Motion and Conservation of Energy

1. Introduction

In this experiment, we use the trajectory equations of a body in two-dimensional free fall to predict where a projectile hits the ground.[1] The initial (launch) velocity of the projectile is determined by applying the law of conservation of energy[2] for the projectile traveling through a long bent tube. We compare the predicted and measured location after the fall. This lab should demonstrate the predictive power of applying physical principles correctly, show that predictions correspond to something in the "real world", and provide insight about deciding what is important in making a measurement.

Remark: You must prepare some derivations at home; otherwise, you may have trouble finishing the lab in the time given.

2. Physical Principles

2.1 Conservation of Energy

One of the most fundamental principles of physics requires that total energy be conserved in all physical processes. This principle is sometimes difficult to apply since there are many different kinds of energy (potential energy, kinetic energy, rotational energy, heat, chemical energy, and mass[3]), and energy can be transformed from one kind to another. We will deal primarily with the first three types of energy, and with a small loss due to friction (which usually ends up converted into heat). In the previous lab you saw how conservation of momentum is sometimes more useful than conservation of energy; in this lab, we will focus on a situation where conservation of energy is the appropriate law to use.

The kinetic energy of a point particle moving with velocity v, is given by $E_{Kin}= \frac{1}{2} mv^2$. Because the rolling ball used in this experiment is not a point object, we need to take into account its rotational motion. This is discussed in the next section.

The potential energy for an object near the earth's surface is given by $E_{Pot}=mgh$, where h is the height above an arbitrarily chosen level. This means that when an object drops from a vertical height h_2 to h_1, $\Delta h=h_1-h_2$, the loss in potential energy is $mg\Delta h$.

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[1] See Sections 4-5, 4-6 of Fundamentals of Physics, 8^{th} Ed., by Halliday, Resnick & Walker.
[2] Chapter 8 of Halliday, Resnick & Walker.
[3] This is the content of Einstein's famous formula: $E=mc^2$!

Some energy is always lost due to friction. If we label the friction energy loss W, it follows that:

$$\textit{Gain in K.E.} = (\textit{P. E. lost}) - W$$

2.2 Estimating the Friction Loss

If there were no friction, all the loss in potential energy ($mg\Delta h$), as the ball rolls from the release point to the launch point, would be converted to kinetic energy. However, because of friction, some of the energy will be dissipated. We need a technique to determine the energy lost to friction. Here, we find the orientation of the track such that that when released from the top, the ball just comes to rest at the lower end of the track. When the ball comes to rest, we know that the kinetic energy is zero, so the difference in potential energy between the initial and final positions must have all been lost to friction. Then, we determine the vertical distance traveled by the ball $\Delta h'$ and $mg\Delta h'$ equals the energy lost to friction. If we assume that the same amount of energy is lost to friction when the track is tilted more steeply, we can use $W=mg\Delta h'$ for all tilts. It follows that:

$$\begin{aligned} \textit{Gain in K.E.} &= (\textit{P. E. lost}) - W \\ &= mg\Delta h - mg\Delta h' \\ &= mg(\Delta h - \Delta h') \end{aligned}$$

<u>*Remark*</u>: There are three types of ball in the experiment: steel (or brass), aluminum and plastic. You have to measure the friction separately for each ball!

2.3 Kinetic Energy including Rotation

In this experiment, we deal with a rolling ball. We need to account for the rolling motion, so that there are two contributions to the kinetic energy:

Kinetic Energy = Kinetic Energy of center of mass + Kinetic Energy of rotation

This relationship takes the analytic form:

$$E_{Kin} = \tfrac{1}{2}\, m \cdot v^2 + \tfrac{1}{2}\, I \cdot \omega^2,$$

where m is the total mass of the object, v is the velocity of the center of mass, I is the moment of inertia ($I = 2/5\ m \cdot R^2$ for a sphere), and ω is the angular velocity of rotation. (For a rolling ball that is not sliding, $\omega = v/R$). The total kinetic energy of the rolling sphere is:

$$E_{Kin} = \frac{1}{2}m \cdot v^2 + \frac{1}{2} \cdot (\frac{2}{5}m \cdot R^2) \cdot (\frac{v}{R})^2$$
$$E_{Kin} = \frac{1}{2}m \cdot v^2 + \frac{1}{5}m \cdot v^2$$
$$E_{Kin} = \frac{7}{10}m \cdot v^2.$$

As can be seen from the final result, the kinetic energy of a rolling ball is slightly larger than that of a point object travelling at the same speed.[4]

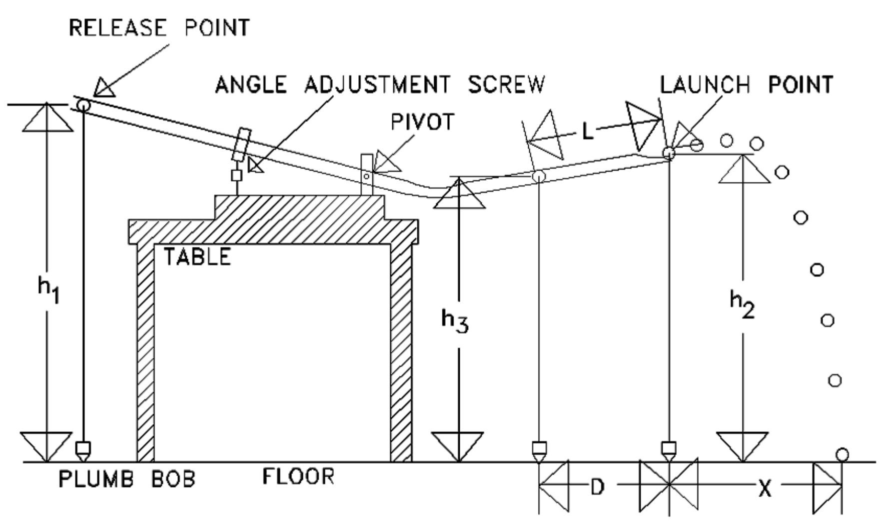
2.2 Parabolic Trajectory

The motion of a mass launched into free fall with initial velocity, *v*, at an angle φ relative to the horizontal, can be treated most easily by evaluating the horizontal (or *x*) and vertical (or *y*) position in terms of the time (*t*) as two independent motions. This problem, in which explicit expressions for *y(t)* and *x(t)* are obtained has been treated in your text and in lecture. (We neglect air resistance while the ball is in free fall.)

3. Experiment

The figure shows the apparatus to be used in this experiment. A ball is released into a tube at the release point, rolls through the tube and emerges at the launch point. You should measure the parameters shown explicitly in the figure, and the additional parameter, $\Delta h'$, which is used to estimate friction losses.

Remark: If you want to clean the tube beforehand, there should be a swab on a string available.



[4] If you have not yet had rotations in lecture, you may not follow this argument in detail. But the last equation indeed does express the kinetic energy of the rolling ball (see HRW Section 11-3). Be sure to use it.

3.1 Prediction of Position

Before you do the experiment, you should derive a set of equations at home that predict the x-position where the ball hits the ground ($y=0$). This expression should depend only on the measured parameters shown in the figure: h_1, h_2, h_3, D, L, as well as the value of $\Delta h'$. You will need to substitute these quantities rather than parameters we do not directly measure, like v or φ.

Rather than derive a single complicated formula for x in terms of symbols for all the preliminary measurements, it is more convenient to calculate, in sequence, several intermediate quantities and then combine them to find x. Bring the sheet with your derivation of the formulae for x in terms of the measured parameters. You should prepare this before coming to the lab! It should be attached to the report when you are finished.

1. Find v, the magnitude of the launching velocity, by using the conservation of total mechanical energy (incorporating the estimate of the energy lost to friction).
2. Find v_x and v_y, the horizontal and vertical components of v, by referring to the geometry of the final section of the track.
3. Find t, the time the ball is in the air, by considering the vertical motion involving v_y and h_2 alone.
4. Finally, find x.

Steps 3 and 4 may be combined by using the trajectory equation $y(x)$, obtained by eliminating the time in the equations for $y(t)$ and $x(t)$.

Choose the heavy metal ball to be your first ball. Adjust the screw such that the ball, when released at the release point, <u>just makes it to the launch point</u> before reversing direction.

- Record h_1' and h_2'.

Increase h_1 with the adjustment screw so that the ball will be launched. Make sure that that h_1-h_2 is at least twice as big as h_1'-h_2'.

- Measure all the required quantities and predict where the ball will hit the floor. Place a coin at that position. Release the ball and see if the ball hits the coin.

Repeat the experiment for the same ball with a different height h_1. (Check or measure all relevant quantities.)

- Comment on your results

Repeat the same steps for the plastic ball, then the aluminum ball.

- The difference $\Delta h' = h_1' - h_2'$ provides a measure of the energy lost to friction as the ball traverses the tube. Order the measurements of the balls from highest friction to lowest friction. Explain why you might have expected this order.

- Which ball would you have expected to fly the furthest horizontal distance from the same release point? Why?
- What are the major sources of error? How far off would your results be if you had not corrected for the friction losses in the tube? From the comparison of your results with the predictions, how much effect might the neglected air resistance in the free-fall trajectory have contributed?

3.2 Quantitative Measurements and Uncertainty

In this part of the experiment, you will take measurements of the hit position relative to that predicted. These data will permit a measure of the spread, or uncertainty, from the reproducibility of the results. You will take measurements for two balls (heavy metal and plastic) for the same orientation of the launching tube.

Place a sheet of white paper on the floor centered at the predicted location and place a piece of carbon paper on top of it. Tape them to the floor. Before proceeding, make a guess of how large the spread of results will be!

Roll a single ball about twenty times using the same setup; you should obtain a number of points marked on the white paper. These should be spread around the expected value. Do this experiment with different papers for the heavy metal ball and for the plastic ball.

- Describe and compare the two spreads. (How large is the spread? Is it uniform in all directions? Should it be? Is the spread the same for both balls? Should it be? Is the spread about as big as you expected?)
- If the spread much bigger along the direction of the trajectory than perpendicular to it, how would you interpret this?
- Make a few suggestions for how you could have improved the first part of the experiment so that you would always hit a smaller area, like that of a dime!

3.3 Checking Formula

If you want to check if your derived formula for the position where the projectile hits the ground is correct, you can use the following data:

h_1	=	124.3 cm	h_3	=	110.0 cm
h_1'	=	122.7 cm	D	=	27.7 cm
h_2	=	119.3 cm	L	=	29.2 cm
h_2'	=	120.5 cm			

The position the projectile hits the ground is then at: x = 30.5 cm.

4. Lab Preparation Examples

Trajectories:

1. What is your predicted value for x, assuming you measure the following values:

$h_1' = 105$ cm	$h_2' = 90$ cm	
$h_1 = 120$ cm	$h_2 = 80$ cm	$h_3 = 65$ cm
$L = 25$ cm	$D = 20$ cm	

2. If $h_1' = 125$ cm and $h_2' = 100$ cm, what percentage of the potential energy is lost to friction?

3. You shoot a bullet with velocity v and an angle φ to the horizontal direction and observe where it hits the ground. You now increase the angle with which a bullet is shot. Does it reach further or not? Does the answer depend on what the original angle was?

Spread:

4. How do you expect the spread of measurements to appear? (Qualitatively, not quantitatively!)

5. Assume you cannot control the release point very well. (In the experiment we control this quite well.) Suppose you sometimes release the ball further up the tube and sometimes further down. How do you think this would affect the spread?

6. Assume you perform the lab outdoors, where there is a strong and unsteady wind blowing along the length of the launch tube. How will your spread look in this case?

Air Resistance:

7. We need to measure the friction in the tube in this experiment. But we do neglect the air resistance of the ball after it is launched. How big an effect do you estimate this to be?

8. Assume you want to make a parachute jump. You know that you can change the air resistance by giving the air a bigger or smaller cross section to resist. But you are curious about what influence your body weight has on the maximum speed you can reach:

   You know that the driving force pulling you down to earth is $m \cdot g$. Furthermore you also know that the air resistance is a function that increases with increasing velocity (and area). (If you are not convinced of this, drive your car at different speeds and put your hand out of the window!) Finally you remember that the maximum speed is the speed when there is no more acceleration (i.e. the force pulling you down and the force pulling you up are equal). With all this knowledge, try to explain why a heavy body can reach a higher maximum velocity than a light one (given that they have the same shape and size). (If you want to use an equation for the air resistance you can e.g. use $F(v) = \text{const.} \cdot v^2$ which is true in some limit.)

Experiment 1-9

Standing Waves

1. Introduction

Waves surround us. Your radio receives signals by means of electromagnetic waves and emits waves that sound (like music) when they hit your ear. Waves are visible on the surface of a body of water. The human body creates waves (like heartbeats) and diagnostics of the body's physical condition involve many different kinds of wave phenomena. The natural world is full of waves, and technology has multiplied their importance to us.

All waves share certain physical similarities. In this experiment, we gain experience with properties shared by many different kinds of waves.[1]

2. Theory

2.1 Waves

Waves are periodic disturbances propagating in space and time. In this lab, we will illustrate properties of waves using waves in a stretched string and sound in an air column.

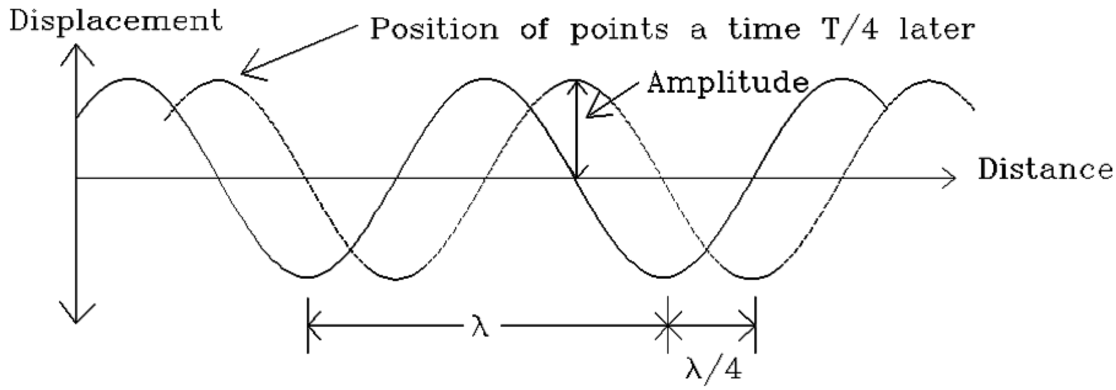
To be able to deal with waves we introduce a number of definitions:

- The period T is the time it takes for a wave to repeat itself.
- The frequency f, defined as $f = 1/T$, measures the number of complete cycles the wave repeats in one second. The unit of frequency (1/seconds) is Hertz (Hz).
- The wavelength λ is the spatial separation between repeating points in a wave.
- The amplitude A of the wave is the maximum magnitude of the displacement.

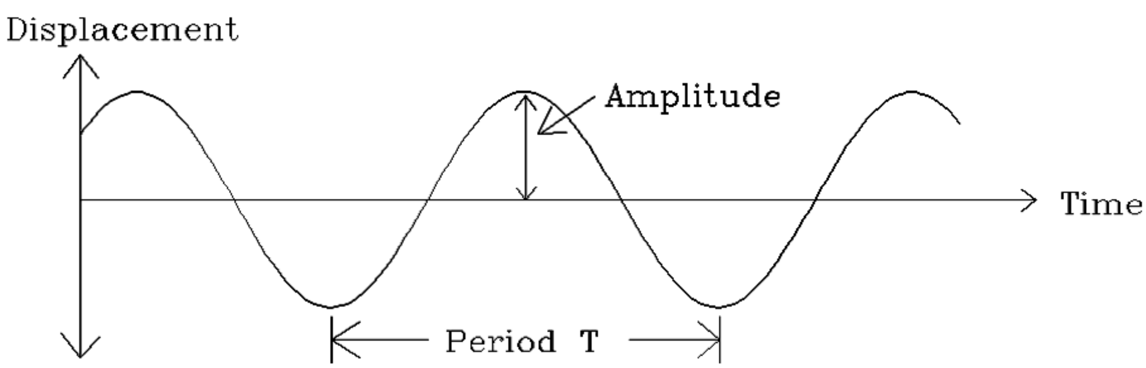
If a long, taut horizontal string is sharply pulled up at some point and released, this part of the string will vibrate up and down. Neighboring points will then follow the motion, and the original disturbance will propagate down the string as a "traveling wave". Since the individual particles vibrate in a direction perpendicular to the direction of propagation, this wave is called a transverse wave. If the disturbance is periodic, i.e., repeated continuously, a wave train will move down the string. The following figure shows the disturbance along the string at a single instant of time, in this case for a sinusoidal wave. If the wave is moving to the right and a second picture is taken a quarter period, T/4, later, all points on the wave will have moved an equal distance to the right, as shown by the dotted curve. The frequency, f, of the wave is defined as the number of times per second

[1] A general discussion of waves is treated in Chapter 16 of Fundamentals of Physics, 8th Ed., by Halliday, Resnick & Walker. See Section 16-13 for standing waves. Electromagnetic Waves are covered in lecture next semester. This experiment should familiarize you with the phenomenology of waves.

the disturbance is repeated (thus $f = 1 / T$). Note that the period of the motion is determined by the cause of the initializing disturbance. During each period T, the wave travels a distance of one wavelength, λ; therefore the velocity of the wave is given by[2] $c = f\lambda$.



The figure below shows the vertical displacement of the string versus time at a fixed location along the string. The time interval between successive identical displacements of a given point is the period T of the wave. Remember that the wavelength of a traveling wave can only be determined when one observes the displacement as a function of distance at one instant in time. The period, on the other hand, is obtained when one observes the displacement of one point as a function of time at one location.



There are, in general, two types of waves: transverse and longitudinal. If the oscillation is perpendicular to the direction of propagation of the wave, then the wave is transverse.[3] If a wave changes along the direction of propagation, it is called a longitudinal wave.

[2] In all the cases here, *c* will be a constant. But in general *c* could be a function of the frequency. (This effect is called dispersion).

[3] The two independent possibilities for making a transverse wave are referred to as the two possible polarizations of the wave. (If the wave oscillates diagonally, it can be viewed as a combination of vertical and horizontal polarizations.)

An example of a longitudinal wave is a sound wave, in which there are periodically changing regions of low and high air pressure along the direction of wave propagation. A sound wave is normally initiated by a vibrating solid (such as a tuning fork), which alternately compresses and rarefies the air adjacent to it. The wave thus consists of pressure variations in the air that is moving away from the fork. Since these variations oscillate along the direction of wave propagation, the sound wave is a longitudinal traveling wave.

The definitions of f, λ , and T, explained above for transverse waves, hold equally for longitudinal waves. The diagrams in the above figures also apply, so long as we understand “displacement” to mean the longitudinal (forward or backward) pressure or density variation of air from its undisturbed equilibrium value. The relationship between wave velocity and the other parameters is also still valid:

$$c=f\cdot\lambda$$

2.2 Standing Waves

So far we have considered only very simple wave disturbances. More complicated waves are created when two or more traveling disturbances are present simultaneously in the same medium. There is a law of superposition which states that the position of a given point on the medium is determined by the sum of all the different disturbance effects. In general, any number of waves can be combined to give a more complicated wave.

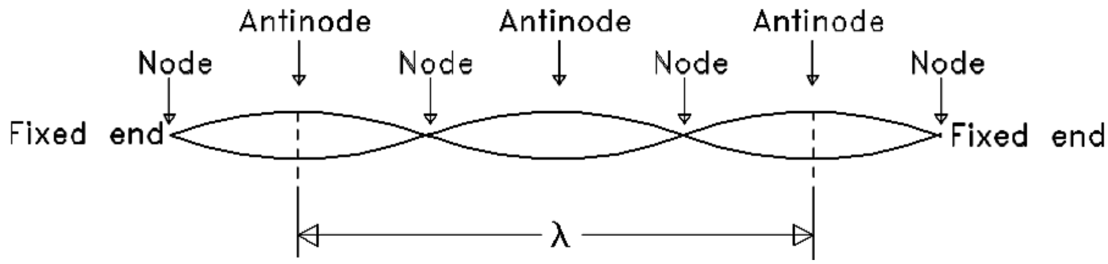
A particularly interesting example occurs when two waves of equal λ, f, and A are traveling along a taut string in opposite directions. (This occurs if the wave encounters a barrier that reflects the wave back in the original direction while the original wave is still propagating.) At some particular points on the string, the two waves will always be out of phase, one wave will try to move the point up and other wave will try to move the point down. The result is that the two waves will cancel each other at this particular point, and so that point will remain stationary. This condition is known as destructive interference, and the points at which this occurs are called nodes.

At other points on the string, the two waves will move up and down together so that the amplitude of the disturbance at these points will be twice what it would be if only one wave were present. This condition is known as constructive interference and the points at which this occurs are called antinodes.

As long as the λ, f, and A of the waves remain fixed, the positions of the nodes and antinodes will not change. The pattern produced in this circumstance is called a standing wave, since it looks as if it is stationary, although it is actually the sum of two waves traveling in opposite directions. The analytic description for the displacement versus position and time with one end fixed at x = 0 is given by:

$$A\cdot\sin[\frac{2\pi}{\lambda}x]\cdot\cos[2\pi f\ t].$$

A string with both ends fixed can be excited with standing waves as shown in the figure below. The fixed ends of the string cannot move, so the string must have nodes at these points. It is evident that the distance between the node at a fixed end and the first anti-node is $\lambda/4$; and the distance between successive nodes (or antinodes) is $\lambda/2$.



The positions on the string where $A \cdot \sin[2\pi/\lambda \cdot x] = 0$ correspond to a node, so whenever $2\pi/\lambda \cdot x$ is a multiple of π, i.e.:

$$\frac{2\pi}{\lambda} x = n \cdot \pi ,$$

(x is a multiple of $\lambda/2$), we find a node. Midway between two nodes we will always find an antinode, a point where the string oscillates maximally.

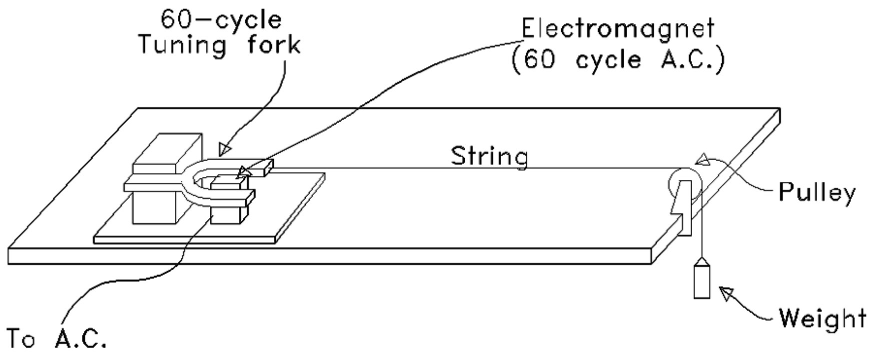
Standing waves can be created in a tube of gas using sound waves. In this part of the experiment, a tuning fork vibrates over the open end of a tube containing air. The tube is sealed at the other end by the water. The motion of the fork causes longitudinal sound waves to travel down the tube. The sound wave is reflected back up by the air-water interface and the incident and reflected waves interfere with each other. The air at the closed end of the tube is cannot move freely, so in order for a standing wave to be produced, a node must exist at the there. The air in the open end of the tube is free to move; so when a standing wave is produced in the tube, the open end is an antinode.

Since the distance between a node and the nearest antinode in a standing wave pattern is $\lambda/4$, it should be evident that the shortest tube in which a standing wave can be established has a length of $\lambda/4$. A standing wave with wavelength λ can be established in longer tubes. All that is required is that a node exist at the closed end and an antinode at the open end – i.e. that the length of the air column is an odd multiple of $\lambda/4$. When a standing wave is produced in the tube, a resonance condition is established and the intensity of the sound will increase.

3. Experiments

3.1 Standing waves on a String

The first part of the experiment deals with transverse waves on a string. A long horizontal string is attached to the tine of a driven tuning fork that vibrates at f = 60 Hz. The other end is fixed at a point where it passes over a pulley. You can change the distance between the pulley and the tuning fork by shifting the base of the tuning fork and you can hang weights of mass M on the end of the string (which produces a string tension $T=M \cdot g$). The apparatus is pictured in the figure below:



Your task is to measure the wavelengths resulting from standing waves produced for various values of the tension. In the previous section, it was stated that the distance between two nodes in a standing wave is $\lambda/2$. So the wavelength is double the distance between adjacent nodes.

Measure the mass and length of the string. Calculate μ, the linear density of the string. Attach the string to the screw on the tuning fork and place it over the pulley. Put a mass of 100 g on the end of the string and choose the distance between pulley and tuning fork such that you get a standing pattern of nodes and antinodes.

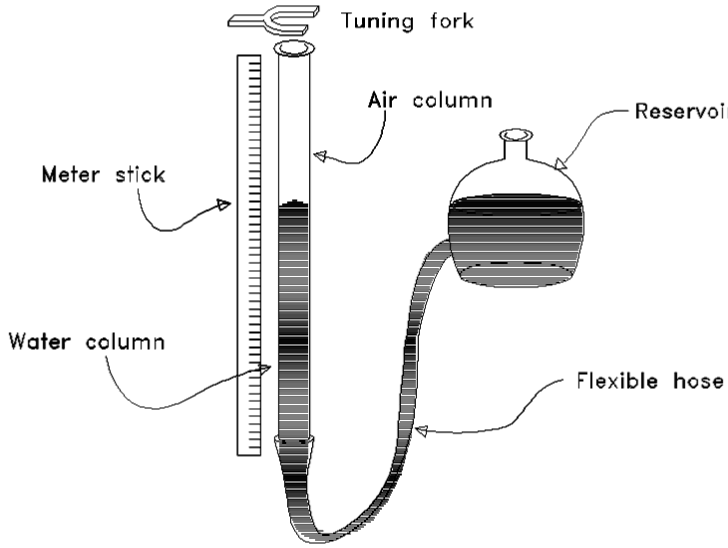
- Measure the wavelength of the standing waves. You get the best results if you measure nodes in the middle of the string and if you average over several measurements.
- Calculate the velocity (including uncertainty) with which the wave propagates on the string, using the relation between frequency and wavelength.
- Analytically calculate the velocity (including uncertainty) of propagation of waves on a string from the physical properties of the string and the following formula:

  $$c = \sqrt{\frac{T}{\mu}},$$

  where T is the tension, and μ is the mass per unit length of the string. Compare your experimental results with this prediction.
- Do this measurement/calculation for 3 different weights.

What are the main sources of error in this experiment?

3.2 Standing Sound Waves



In the second part, we measure the wavelength of standing longitudinal waves.

Sound waves (longitudinal waves) are set up in a long open tube, which is partly filled with water, as shown in the figure to the right. We change the height of the water column by lifting or lowering a water reservoir. Produce a sound wave using a tuning fork. There are two tuning forks, a 512 Hz one and a 1024 Hz one. You should perform the experiment with both of them. Strike the tuning fork and hold it at the top of the glass tube. As you change the level of water in the tube, you find some water levels at which the sound from the tuning fork becomes significantly louder. This occurs whenever you have created a standing sound wave in the glass tube. By measuring the distances between water levels that achieve successive sound maxima, you can determine the wavelengths of the sound waves.

As before, from the wavelengths of the standing waves and the frequencies, you can compute the velocity of propagation of the sound waves for each case. This should equal the speed of sound in air.

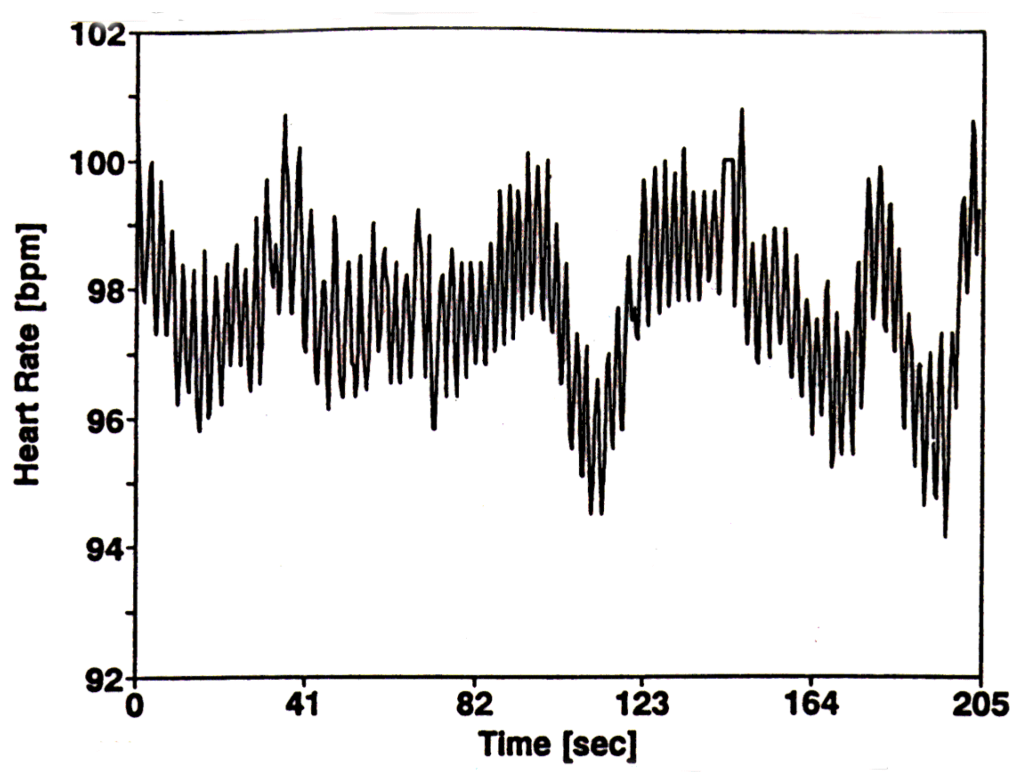
Remark: It is often easiest if you lower the water level continuously and hit the tuning fork somewhat hard. However, please be careful and don't damage any equipment.

- Measure the wavelength for both tuning forks. There should be small string-rings on the glass tube that you can use to mark the levels at which you get resonance (standing waves). To improve your data try to average over several measured values. Also don't forget to include uncertainties.
- Calculate the speed of sound for both frequencies. Do you get the same value within uncertainty? Is your value for the speed of sound close to the standard value of 340 m/s? What could be reasons that you get a different value?
- Was one of the tuning forks easier to hear than the other? If yes, do you have an idea why?
- Give the main sources of error and make suggestions of how the setup could be improved.

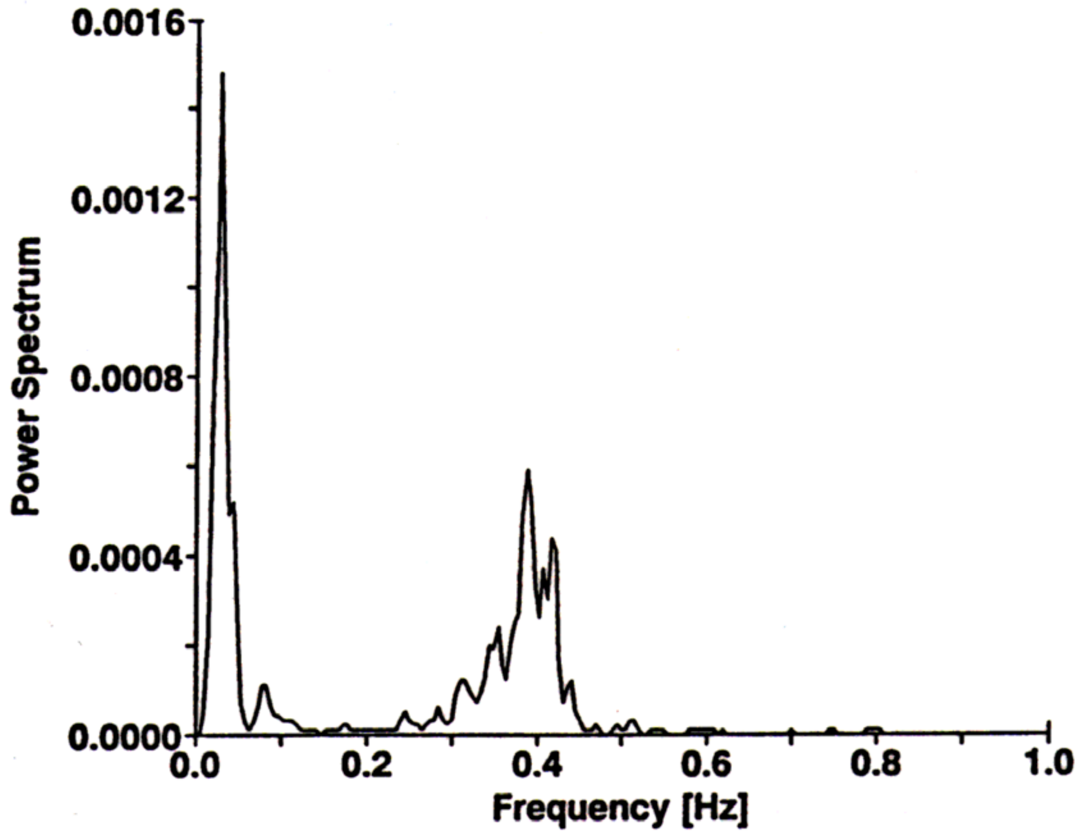
4. Applications

Biological systems that create periodic pulses (like a beating heart) are optimized for a range of operating conditions. The variations in the operating parameters often provide an important diagnostic tool. For example, the period of beats in a human heart can vary; one may ask to what extent and how often the heartbeat rate changes. To identify patterns of heart rate, one makes use of a mathematical procedure called Fourier analysis. Besides the dominant frequency of the heartbeat (about 1 Hz), there are other frequencies that correspond to changes in the heart rate; Fourier analysis uses the heart rate to make a diagram of the amplitudes of the lower frequencies present in the heart rate.

This may at first sound a little bit complicated, but the basic idea is simple. In the experiment, we find only specific standing waves for each system. With the system parameters fixed, only certain frequencies are possible: most frequencies cannot achieve a standing wave. Obviously these frequencies are special and characterize the system! If you look now at a complicated oscillation of the system and take the Fourier spectrum, you find that only these special frequencies contribute. Other frequencies don't contribute at all. A beating human heart is, of course, much more complicated than the systems we deal with in the lab, but the basic ideas are the same!



This picture shows the heart rate of a healthy adult over time. As one can easily see, the heart rate changes over time. The frequencies with which the heart rate changes can most easily be seen in the Fourier spectrum (sometimes called power spectrum) in the next figure.



One immediately sees a prominent change in the heart rate with a frequency of about 0.4 Hz. Therefore, roughly every 2.5 s the heart rate of this person changes. This frequency corresponds to the many little spikes we saw in the first diagram. (Another characteristic frequency is at about 0.02 Hz, corresponding to a change about every 50 s. We will not deal with this part here, even though it contains valuable information.)

Where does the change every 2.5 s come from? It turns out that the person took a breath about every 2.5 s. The heart then speeds up slightly to pump more blood through the vessels of the lung, where the blood is oxygenated. Subsequently, it slows down again. (This particular frequency is called the respiratory frequency.)

The process of speeding up and slowing down is governed by the autonomic nervous system. Some diseases (e.g. diabetes) can damage the autonomic nervous system and therefore stop this adaptation of the heart rate. For example, as diabetes reaches its final stage the peak in the Fourier spectrum at the respiratory frequency vanishes.

This method has the nice feature that it is non-invasive, relatively simple, and shows dynamic processes rather than snapshot pictures.

Reference: Amos D. Korczyn: Handbook of Autonomic Nervous System Dysfunction.

5. Lab Preparation Problems

Waves:

1. Is a wave on the water surface a longitudinal or transverse wave?

2. The wavelength of visible light is between 400-800 nm. What is the frequency of visible light?

Standing Waves:

3. A transformer is humming at a frequency of 60 Hz and produces a standing wave. What is the distance between adjacent nodes? What is the distance between adjacent antinodes?

4. You have a string and produce waves on it with 50 Hz. The wavelength you measure is 7 cm. What is the speed of the wave on this string?

5. You put a mass of 400 g on the string of experiment 1. (The string is 50 cm long and weights 12.5 g) What distance between adjacent nodes do you then expect for a frequency of 100 Hz. (Use $g = 10\ m/s^2$)

6. With a 660 Hz tuning fork you measure a distance of 25 ± 2 cm between adjacent nodes. Is the value of $c = 340$ m/s within the uncertainty of your measured value?

Explanations:

7. If you blow air along the top of an open soda bottle you can excite a standing wave in the bottle and you hear a sound. Explain what happens if you put some water into the bottle and then perform the same experiment!

Experiment 1-10

Specific Heat and Mechanical Equivalent of Heat

1. Introduction

James Joule (1818-1889) was the first to establish by experiment that heat and mechanical work are both forms of energy and that they can be quantitatively related to one another. The purposes of this experiment are:

1. to measure the specific heat of a substance (the amount of heat that must be applied to a unit mass of substance in order to change its temperature by one degree) and
2. to determine by experiment the relationship between “heat” energy units (MKS - kilocalorie) and “mechanical” energy units (MKS - Joule).

Part I - Specific Heat

I. 2. Physical Principles

The amount of heat required to raise the temperature of a unit mass of a substance by one degree is called the specific heat of that substance. Thus, if an amount of heat Q is added to a substance of mass, m, having a specific heat, c, the change in temperature, $\Delta T = T_f - T_i$, will be given by:

$$\Delta T = \frac{Q}{mc} \text{ or } Q = mc\Delta T$$

In the MKS system, the unit of heat is the kilocalorie and it is defined so that the specific heat of water is one – i.e. when one kilocalorie of heat is applied to one kilogram of water, its temperature will rise by one degree Celsius.

When two or more substances at different temperatures are brought together, they will reach thermal equilibrium after a period of time because heat will flow from the warmer to the cooler substance until they are both at the same temperature. If the materials which make up the system of interest are insulated so that no heat can be transferred to or from the surroundings, the process is said to be adiabatic. Since heat is a form of energy, the law of conservation of energy requires that for an adiabatic process the sum of all the heat transfers among parts of the system must be equal to zero, since no heat can be lost to or gained from the surroundings. Note: If heat is added to a substance, then $T_f > T_i$ and Q is positive; if heat is removed, then $T_f < T_i$ and Q is negative.

In this part of the experiment, a mass m_L of lead will be heated to a temperature T_L. The hot lead will be placed in a mass m_W of water at a known temperature T_W. Since $T_W < T_L$, heat will flow from the lead into the water until the equilibrium temperature T_e is reached. Then the sum of the heat transfers will equal zero:

$$Q_L = m_L c_L (T_e - T_L) \qquad Q_L + Q_W = 0$$
$$Q_W = m_W c_W (T_e - T_W)$$

The specific heat c_L of lead can be determined from the known values of temperature and mass. (The specific heat of water, c_W, is 1 kCal/ kg deg C.)

I. 3. Experimental Apparatus

The experimental apparatus consists of the following items:

1. A block of lead held by a string.
2. An electrically heated kettle, in which you can bring water to a boil and then, putting the block of lead into the boiling water, heat the lead to the same temperature. (Do not fill the kettle more than 2/3 full.)
3. A "Styrofoam" container of negligible mass which serves to insulate the small amount of cool water and the hot block of lead and allow them to come to thermal equilibrium. (Put a paper cup inside the Styrofoam cube before adding water. Do not put water directly into the cube.)
4. A thermometer for measuring the temperature of the water in the container.
5. A balance for weighing the water and the lead.

I. 4. Procedure

Use the above equipment to measure c_L for two blocks of lead.

- Compare your result with the accepted value for the specific heat of lead and comment on sources of error in the experiment:

$$c_L = 0.031 \frac{\text{kCal}}{\text{kg deg C}}$$

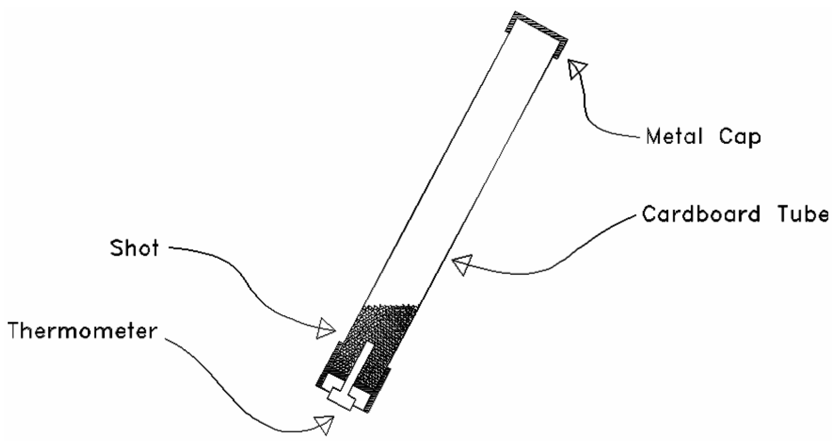
- Why is it advisable to start with water which is a few degrees below room temperature?
- It is recommended that you stir gently to assure uniform temperature in the water. Does stirring introduce a significant source of error?

Safety Caution: Unplug the water heater when you are done, empty all the water out, and leave the heater disconnected.

Part II - Mechanical Equivalent of Heat

II. 2. Physical Principles

When an object starts with zero initial velocity and falls through a distance *h*, its initial potential energy *mgh* is converted into kinetic energy as it falls. When the object comes to rest after impact at the bottom of the fall, the kinetic energy is converted into the energy of molecular vibrations (heat). This fact is apparent from the observable rise in the object's temperature after the fall. When the specific heat and the mass of the object are known, the amount of heat added to the object during impact can be determined from the change in temperature. If you are sure that all of the original mechanical energy was transformed into heat energy and that only a negligible amount of heat escaped from the object, you can determine the relationship between the units used for mechanical energy and the units used for heat energy. This conversion constant is called the mechanical equivalent of heat, ***J***.



II. 3. Experimental Apparatus

In this experiment, about 1 kg of lead "shot" (small pellets) has been placed in a cardboard tube about one meter long, which is permanently closed at both ends, with a metal dial thermometer mounted in one end. The cardboard is an extremely poor conductor and has a low heat capacity. Thus, when the shot is made to fall vertically from one end of the tube to the other a large number of times, most of the heat developed is absorbed by the shot itself, while an insignificant portion is taken up by the cardboard and the light thermometer stem. The mechanical equivalent of heat can be determined by measuring *h*, the total distance through which the shot has fallen, and ΔT, the temperature rise of the shot, and by using the value of the specific heat of lead measured in Part I.

Safety Caution: Do not attempt to remove either of the caps from the cardboard tube; they are meant to be permanently sealed (this prevents the spilling of any lead dust, which accumulates in the tube.)

II. 4. Procedure

Cool the lead shot 5-7 degrees below room temperature by placing the end of the cardboard tube for a few minutes in an empty metal pipe which is suspended in an ice bath. Do not get the tube wet! Agitate the shot very slightly to ensure thermal equilibrium. Allow the thermometer to remain in contact with the shot for 2-3 minutes before taking a reading

- After thermal equilibrium has been established, record the temperature of the shot. It should be about 3-4 degrees below room temperature.

Proceed to invert the tube N times (approximately 75) in rapid succession. When inverting the tube, place the lower end on the table so that the falling shot does not force out the ends. Try to keep your hands away from the ends of the tube so that they do not absorb the heat of your hands.

When you have completed the last inversion, repeat the procedure for recording the temperature of the shot.

- For best accuracy the temperature of the shot should be approximately as much above room temperature as it was below room temperature initially. Why? If this is not the case, you should discuss the error that this would introduce in your conclusion.

Measure the total height h through which the center of mass of the shot has fallen. (Use the marks on the outside of the tube, which indicate the location of the shot at each end of the tube.)

- Using the value of the specific heat of lead measured in Part I, and the measured value of h and ΔT, calculate J, the mechanical equivalent of heat. Recall that:

$$J = \frac{W}{Q} = \frac{mgh}{mc\Delta T} = \frac{gh}{c\Delta T}$$

- Repeat the measurement, and compare the average value of J from your two trials with the accepted value of 4185 Joules/kCal. What are the major sources of error in your measurement?