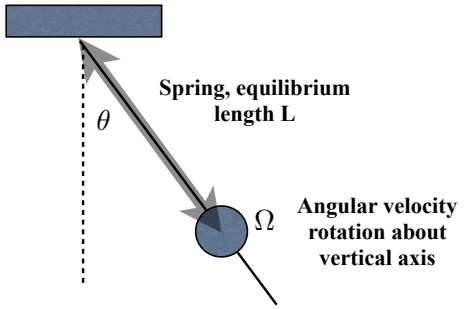
PHY W3003: Practice Midterm II

1 (20 points) A particle of mass m slides without friction along a rigid rod which rotates about a vertical axis with fixed polar angle θ with *time-varying* azimuthal angular velocity $\Omega(t) = A\, cos\Omega_1\, t$. The particle is connected to the top of the rod by a spring of force constant k and equilibrium length L. (See Figure).



(a) (5 points) Please write down the Lagrangian of this system using as dynamical variable the length l of the spring.

(b) (5 points) Please find the equation of motion

(c) (10 points) For some ranges of Ω_1 and A the motion may become unbounded (extension of the spring can become arbitrarily large). Please find these ranges

2 (20 points) A particle of mass m moves in three dimensions subject to the potential $U(x, y, z) = \frac{k}{2}\left(x^2 + y^2\right)$.

(a) (5 points) Please state whether energy and the different components of momentum and angular momentum are conserved and *give your reasons.*

(b) (10 points) Suppose that the particle is subject to a frictional force $\vec{F} = -\gamma\vec{v}$, and that at time $t = 0$ the particle is at position $x = x_0$, $y = 0$ and $z = 0$ with velocity $\vec{v} = v_z\hat{z}$. Please find the motion at all subsequent times

(c) (5 points) Determine how much work the particle does on the degrees of freedom providing the friction over the entire time period $t = 0$ to $t = \infty$. (*Note that this answer can be obtained even if you dont have the full solution requested in (b).*

3 (20 points) A particle of mass m moves in three dimensions subject to the potential

$$U(r) = \frac{U_8}{8r^8} - \frac{U_5}{5r^5}$$

with U_8 and U_5 both positive.

(a) (10 points) Please determine the range of angular momenta for which circular motion is possible and the radius and period of the resulting orbit.

(b) (5 points) Suppose the motion of a particle moving in the circular orbit found in (a) is slightly perturbed, so that at time $t = 0$ the value of the radial coordinate is changed to $r \to r_{circ} + \delta$ with δ small, but no velocities are changed. Please determine, in terms of δ, U_8, U_5 r_{circ} and m the maximum value of the radial velocity $\dot{r}$. *for this problem you may work to lowest order in δ) and you dont need to simplify the expressions.* (c) (5 points) Is the resulting orbit closed? (Give reasons).