CODATA recommended values of the fundamental physical constants: 1998*†‡§

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This paper gives the 1998 self-consistent set of values of the basic constants and conversion factors of physics and chemistry recommended by the Committee on Data for Science and Technology (CODATA) for international use. Further, it describes in detail the adjustment of the values of the subset of constants on which the complete 1998 set of recommended values is based. The 1998 set replaces its immediate predecessor recommended by CODATA in 1986. The new adjustment, which takes into account all of the data available through 31 December 1998, is a significant advance over its 1986 counterpart. The standard uncertainties (i.e., estimated standard deviations) of the new recommended values are in most cases about 1/5 to 1/12 and in some cases 1/160 times the standard uncertainties of the corresponding 1986 values. Moreover, in almost all cases the absolute values of the differences between the 1998 values and the corresponding 1986 values are less than twice the standard uncertainties of the 1986 values. The new set of recommended values is available on the World Wide Web at physics.nist.gov/constants.

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TABLE I. Some exact quantities relevant to the 1998 adjustment.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>speed of light in vacuum</td>
<td>(c, c_0)</td>
<td>299 792 458 m s(^{-1})</td>
</tr>
<tr>
<td>magnetic constant</td>
<td>(\mu_0)</td>
<td>(4\pi \times 10^{-7} \text{ N A}^{-2})</td>
</tr>
<tr>
<td>electric constant</td>
<td>(\varepsilon_0)</td>
<td>((\mu_0 c^2)^{-1} = 8.854 187 817 \ldots \times 10^{-12} \text{ F m}^{-1})</td>
</tr>
<tr>
<td>molar mass of (^{12}\text{C})</td>
<td>(M(\text{^{12}C}))</td>
<td>(12\times 10^{-3} \text{ kg mol}^{-1})</td>
</tr>
<tr>
<td>conventional value of Josephson constant</td>
<td>(K_{\text{J-90}})</td>
<td>483 597.9 GHz V(^{-1})</td>
</tr>
<tr>
<td>conventional value of von Klitzing constant</td>
<td>(R_{\text{K-90}})</td>
<td>25 812.807 Ω</td>
</tr>
</tbody>
</table>

Klitzing constants \(K_{\text{J-90}}\) and \(R_{\text{K-90}}\), and the conventional electric units that they imply.

Section III and Appendices A to D are the most critical portions of the paper because they are devoted to the review of all the available data that might be relevant to the 1998 adjustment. This includes theoretical expressions for bound-state corrections to magnetic moments (Sec. III.C.2.), energy levels of the hydrogen atom (Appendix A), the magnetic moment anomalies of the electron and muon \(a_e\) and \(a_\mu\) (Appendices B and C), and the ground-state hyperfine splitting in muonium \(\Delta \nu_{\text{Mu}}\) (Appendix D).

The experimental data include relative atomic masses of various atoms, transition frequencies in hydrogen, magnetic moment ratios involving various atomic particles such as the electron and muon, values of \(\Delta \nu_{\text{Mu}}\), shielded gyromagnetic ratios involving the proton and the helium (nucleus of the \(^3\text{He}\) atom), values of the Josephson and von Klitzing constants \(K_j\) and \(R_k\), the product \(K_j^2 R_k\), the \(\{220\}\) lattice spacing of silicon \(d_{220}\), the quotient \(h/m_n d_{220}\) (\(m_n\) is the neutron mass), the Faraday and molar gas constants, and the Newtonian constant of gravitation.

In order to keep this paper to an acceptable length, theoretical calculations and experiments are described only in sufficient detail to allow the reader to understand our treatment of them and the critical issues involved, if any. It is left to the reader to consult the original papers for additional details and to understand fully the difficulty of experimentally determining the value of a quantity with a relative standard uncertainty of \(1 \times 10^{-8}\) (one part in 100 million), or of calculating a fractional contribution of \(1 \times 10^{-8}\) to the theoretical expression for a quantity such as \(\Delta \nu_{\text{Mu}}\).

There is nothing special about the order in which the major categories of data are reviewed. It was selected on the basis of what seemed reasonable to us, but a different ordering could very well have been chosen. Similarly, there is nothing special about the order in which we review measurements of the same quantity from different laboratories. Factors that influenced our ordering choice in any particular case include the uncertainty quoted by the experimenters, the date the result was published, and the alphabetical order of the laboratories.

To avoid confusion, we identify a result by its year of publication rather than the year the result became available. For example, if a result was given at a meeting in 1988 but the publication date of the paper formally reporting the result is 1990, the date used in the result’s identification is 1990 rather than 1988.

Section IV gives our analysis of the data. Their consistency is examined by first comparing directly measured values of the same quantity, and then by comparing directly measured values of different quantities through the values of a third quantity such as the fine-structure constant \(a\) or Planck constant \(h\) that may be inferred from the values of the directly measured quantities. The data are then examined using the standard method of least squares, which is described in Appendix E, and based on this study the final input data (including their uncertainties) for the 1998 adjustment are determined.

Section V gives, in several tables, the 1998 CODATA recommended values of the basic constants and conversion factors of physics and chemistry. Included among the tables is the covariance matrix of a selected group of constants, the utilization of which, together with the law of propagation of uncertainty, is reviewed in Appendix F. The tables are followed by a summary of how the 1998 recommended values are obtained from the values of the subset of constants resulting from the least-squares fit of the final input data.

Section VI concludes the main text with a comparison of the 1998 set of recommended values with the 1986 set, a discussion of the implications of some of the 1998 recommended values, the outlook for the future based on work currently underway, and suggestions for future work.

II. SPECIAL QUANTITIES AND UNITS

Some special quantities and units that are relevant to the 1998 adjustment are reviewed in the following sections. Those special quantities with exactly defined numerical values are given in Table I.

A. Speed of light in vacuum \(c\) and realization of the meter

The current definition of the unit of length in the SI, the meter, was adopted by the 17th General Conference on Weights and Measures (CGPM, Conférence Générale des Poids et Mesures) in 1983. It reads (BIPM, 1998)

"The meter is the length of the path traveled by light in vacuum during a time interval of \(1/299\,792\,458\) of a second." This definition replaced the definition adopted by the 11th CGPM in 1960 based on the krypton 86 atom, which in turn replaced the original definition of the
meter adopted by the first CGPM in 1889 based on the international Prototype of the meter. As a consequence of the 1983 definition, the speed of light in vacuum \( c \) is now an exact quantity:

\[
c = 299 792 458 \text{ m/s.} \tag{1}
\]

A number of the experiments relevant to the 1998 adjustment of the constants require an accurate practical realization of the meter. The three ways to realize the meter recommended by the International Committee for Weights and Measures (CIPM, *Comité International des Poids et Mesures*) are (BIPM, 1998) (a) by means of the length \( l \) traveled by electromagnetic waves in vacuum in a time \( t \) using the relation \( l = c \ t; \) (b) by means of the wavelength in vacuum \( \lambda \) of a plane electromagnetic wave of frequency \( f \) using the relation \( \lambda = c / f; \) and (c) by means of one of the CIPM recommended radiations and its stated wavelength or stated frequency. The CIPM published its first list of recommended values of specified radiations in 1983 (called “Mise en Pratique of the Definition of the Meter”), and subsequently issued an improved and extended *Mise en Pratique* in 1992 and again in 1997 (Hudson, 1984; Quinn, 1993; BIPM, 1998).

For experiments requiring the accurate measurement of a length, except for those related to the determination of the Rydberg constant, the changes in the recommended values from one *Mise en Pratique* to the next are well below the uncertainties of the experiments and need not be taken into account. In the case of the Rydberg constant, the changes would need to be taken into account in analyzing data that span the changes in recommended values. However, as discussed in Sec. III.B, the older data are no longer competitive, and in the newer experiments the frequencies of the relevant lasers used were determined in terms of the SI definition of the second based on the cesium atom. That definition is as follows (BIPM, 1998): “The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.”

**B. Magnetic constant \( \mu_0 \) and electric constant \( \varepsilon_0 \)**

The definition of the ampere, the unit of electric current in the SI, reads (BIPM, 1998) “The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to \( 2 \times 10^{-7} \) N/m of length.”

The expression from electromagnetic theory for the force \( F \) per length \( l \) between two straight parallel conductors a distance \( d \) apart in vacuum, of infinite length and negligible cross section, and carrying currents \( I_1 \) and \( I_2 \) is

\[
F = \frac{\mu_0 I_1 I_2}{2 \pi d}. \tag{2}
\]

This expression and the definition of the ampere in combination imply that the magnetic constant \( \mu_0 \), also called the permeability of vacuum, is an exact quantity given by

\[
\mu_0 = 4 \pi \times 10^{-7} \text{ N A}^{-2} = 4 \pi \times 10^{-7} \text{ H m}^{-1} = 12.566 370 614 \ldots \times 10^{-7} \text{ N A}^{-2}. \tag{3}
\]

Because the electric constant \( \varepsilon_0 \), also called the permittivity of vacuum, is related to \( \mu_0 \) by the expression \( \varepsilon_0 = 1 / \mu_0 c^2 \), it too is an exact quantity:

\[
\varepsilon_0 = \frac{1}{4 \pi \times 10^{-7} \text{ N A}^{-2} c^2} = 8.854 187 817 \ldots \times 10^{-12} \text{ F m}^{-1}. \tag{4}
\]

**C. Electronvolt eV, unified atomic mass unit u, and related quantities**

The electron volt eV and the unified atomic mass unit u are not units of the SI but are accepted for use with the SI by the CIPM (BIPM, 1998). Energies and masses of atomic particles are more conveniently expressed in eV and u than in the corresponding SI units of energy and mass, the joule and the kilogram, and in the case of mass, with significantly smaller uncertainties.

One electronvolt is the kinetic energy acquired by an electron in passing through a potential difference of 1 V in vacuum. It is related to the joule by

\[
1 \text{ eV} = (e/C) J = 1.60 \times 10^{-19} \text{ J}, \tag{5}
\]

where \( e \) is the elementary charge and \( e/C \) is the numerical value of the elementary charge when expressed in the unit Coulomb (see Sec. LB).

The unified atomic mass unit \( u \) is \( \frac{1}{12} \) times the mass \( m(^{12}\text{C}) \) of a free (noninteracting) neutral atom of carbon 12 at rest and in its ground state:

\[
1 \text{ u} = m_u = \frac{m(^{12}\text{C})}{12} = 1.66 \times 10^{-27} \text{ kg}, \tag{6}
\]

where the quantity \( m_u \) is the atomic mass constant.

The relative atomic mass \( A_r(X) \) of an elementary particle, atom, or more generally an entity \( X \), is defined by

\[
A_r(X) = \frac{m(X)}{m_u}, \tag{7}
\]

where \( m(X) \) is the mass of \( X \). Thus \( A_r(X) \) is the numerical value of \( m(X) \) when \( m(X) \) is expressed in u, and evidently \( A_r(^{12}\text{C}) = 12 \) exactly. [For particles such as the electron \( e \) and proton \( p \), the symbol \( m_X \) rather than \( m(X) \) is used to denote the mass. Further, for molecules the term relative molecular mass and symbol \( M_r(X) \) are used.]

The quantity “amount of substance” of a specified elementary entity is one of the seven base quantities of the SI, and its unit the mole, with symbol mol, is one of the seven base units of the SI (BIPM, 1998). One mole is the amount of substance \( n(X) \) of a collection of as many
specified entities X as there are atoms in 0.012 kg of carbon 12, where it is understood that the carbon atoms are free, neutral, at rest, and in their ground state. The molar mass \( M(X) \) is the mass of a collection of entities X divided by the amount of substance \( n(X) \) of the collection. Clearly, the molar mass of free carbon 12 atoms at rest, \( M(^{12}C) \), is exactly
\[
M(^{12}C) = 12 \times 10^{-3} \text{ kg mol}^{-1} = 12 M_u, \tag{8}
\]
where for convenience we introduce the molar mass \( M_u \) defined by
\[
M_u = 10^{-3} \text{ kg mol}^{-1}, \tag{9}
\]
so that in general
\[
M(X) = A(X) M_u, \tag{10}
\]
[Mills et al. (1993) use \( M^* \) to represent \( 10^{-3} \text{ kg mol}^{-1} \), but we believe that \( M_u \) is preferable, because it does not require a special font.]
The Avogadro constant \( N_A \approx 6.02 \times 10^{23} \text{ mol}^{-1} \) is defined as the quotient of the molar mass and atomic mass constants:
\[
N_A = \frac{M_u}{m_u}, \tag{11}
\]
or equivalently
\[
N_A = \frac{M(X)}{m(X)}. \tag{12}
\]
For a collection of \( L \) different types of free entities \( X_1, X_2, \ldots, X_L \), the total amount of substance of the collective entity X is given by
\[
n(X) = \sum_{i=1}^{L} n(X_i), \tag{13}
\]
and
\[
x(X_i) = \frac{n(X_i)}{n(X)} \tag{14}
\]
is the amount-of-substance fraction (also called mole fraction) of entity \( X_i \). The mean relative atomic mass of \( X \) is given by
\[
A(X) = \sum_{i=1}^{L} x(X_i) A(X_i), \tag{15}
\]
and the mean molar mass is
\[
M(X) = A(X) M_u. \tag{16}
\]
An example relevant to Sec. III.H is the mean molar mass \( M(\text{Ag}) \) of the silver atoms of a given sample containing the two naturally occurring isotopes \(^{107}\text{Ag} \) and \(^{109}\text{Ag} \). In this case \( M(\text{Ag}) = A(\text{Ag}) M_u \), where
\[
A(\text{Ag}) = x(\text{Ag}) A(\text{Ag}) + x(\text{Ag}) A(\text{Ag}), \tag{17}
\]
and
\[
x(A\text{Ag}) = n(A\text{Ag})/n(\text{Ag}) \]
is the amount-of-substance fraction of the silver isotope of nucleon number (mass number) \( A \).

### D. Josephson effect and Josephson constant \( K_J \), and quantum Hall effect and von Klitzing constant \( R_K \)

This section briefly reviews two truly remarkable quantum phenomena of condensed-matter physics known as the Josephson effect and quantum Hall effect, as they relate to the fundamental physical constants.

1. **Josephson effect**

It is now well known that the ac and dc Josephson effects are characteristic of weakly coupled superconductors, for example, a superconductor-insulator-superconductor (SIS) tunnel junction, or a superconductor-normal metal-superconductor (SNS) weak link [see, for example, the book by Likharev (1986)]. When such a Josephson device is irradiated with electromagnetic radiation of frequency \( f \), usually in the range 10 GHz to 100 GHz, its current vs voltage curve exhibits current steps at precisely quantized Josephson voltages \( U_J \). The voltage of the \( n \)th step, where \( n \) is an integer, is related to the frequency \( f \) by
\[
U_J(n) = \frac{n f}{K_J}. \tag{18}
\]
Here \( K_J \) is the Josephson constant, formerly called the Josephson frequency–voltage quotient, because it is equal to the step number \( n \) times the quotient of the frequency and voltage. [Note that, under certain circumstances, steps that accurately obey Eq. (18) with \( n \) replaced by \( n \pm \frac{1}{2} \) may also be observed (Genevès et al., 1993).]

An impressive body of experimental evidence has accumulated since the Josephson effect was predicted nearly 40 years ago (Josephson, 1962) that clearly demonstrates that \( K_J \) is a constant of nature. For example, with different but small uncertainties, \( K_J \) has been shown to be independent of experimental variables such as irradiation frequency and power, current, step number, type of superconductor, and type of junction [see Refs. 12 to 22 of Taylor and Witt (1989)]. In one experiment (Tsai, Jain, and Lukens, 1983) it was shown that \( K_J \) was the same for two SNS junctions composed of different superconductors (biased on their \( A \) steps) to within the \( 5 \times 10^{-16} \) relative uncertainty of the comparison. More recently, it was shown that \( K_J \) for a weak link of the high-\( T_c \) ceramic superconductor \( \text{YBa}_2\text{Cu}_3\text{O}_7-\delta \) was equal to \( K_J \) for a weak link of Nb to within the \( 5 \times 10^{-8} \) relative uncertainty of the experiment (Tarbeyev et al., 1991).

The theory of the Josephson effect predicts, and the experimentally observed universality of \( K_J \) is consistent with the prediction, that
\[
K_J = \frac{2e}{h} \approx 483,598 \text{ GHz/V}, \tag{19}
\]
where \( e \) is the elementary charge and \( h \) is the Planck constant (Clarke, 1970; Langenberg and Schrieffer, 1971; Hartle, Scalapino, and Sugar, 1971; Likharev, 1986). Some arguments given for the exactness of Eq.
2. Quantum Hall effect

It is also now well known that the integral and fractional quantum Hall effects are characteristic of a two-dimensional electron gas (or 2DEG) [see, for example, the book by Prange and Girvin (1990)]. In practice, such an electron gas may be realized in a high-mobility semiconductor device such as a GaAs–AlGaAs heterostructure or a silicon-metal-oxide-semiconductor field-effect transistor (MOSFET), of usual Hall-bar geometry, when the applied magnetic flux density is of order 10 T and the device is cooled to a temperature of order 1 K. Under these conditions, the 2D electron gas is fully quantized and for a fixed current $I$, and the number of electrons per flux quantum threading the heterostructure, or of fundamental constants. It follows from Eq. (20) that the number of Landau levels fully occupied and equal to the resistance of the first plateau. We confine our discussion to the integral quantum Hall effect transistor (MOSFET), of usual Hall-bar geometry, when the applied magnetic flux density $B$ is of order 10 T and the device is cooled to a temperature of order 1 K. Under these conditions, the 2D electron gas is fully quantized and for a fixed current $I$ through the device, there are regions in the curve of $U_{H}$ vs. $B$ for a heterostructure, or of $U_{H}$ vs gate voltage $U_{g}$ for a MOSFET, where the Hall voltage $U_{H}$ remains constant as $B$ or $U_{g}$ is varied. These regions of constant $U_{H}$ are called quantized Hall resistance plateaus.

In the limit of zero dissipation in the direction of current flow, the quantized Hall resistance of the $i$th plateau $R_{H}(i)$, which is the quotient of the Hall voltage of the $i$th plateau $U_{H}(i)$ and the current $I$, is quantized:

$$R_{H}(i) = \frac{U_{H}(i)}{I} = \frac{R_{K}}{i}, \tag{20}$$

where $i$ is an integer and $R_{K}$ is the von Klitzing constant. (The integer $i$ has been interpreted as the filling factor—the number of Landau levels fully occupied and equal to the number of electrons per flux quantum threading the sample. We confine our discussion to the integral quantum Hall effect because, to date, no experimental work on the fractional quantum Hall effect is relevant to the fundamental constants.) It follows from Eq. (20) that the von Klitzing constant $R_{K}$ is equal to the quantized Hall resistance of the $i$th plateau times the plateau number, and hence is equal to the resistance of the first plateau.

As with the Josephson effect, a significant body of experimental evidence has accumulated since the discovery of the quantum Hall effect nearly 20 years ago (von Klitzing, Dorda, and Pepper, 1980) that clearly demonstrates that $R_{K}$ as defined by Eq. (20) is a constant of nature. To measure this constant accurately, certain experimental criteria must be met. These criteria are given in technical guidelines developed by the CIPM’s Consultative Committee for Electricity and Magnetism (CCEM, Comité Consultatif d’Electricité et Magnetisme, formerly Comité Consultatif d’Electricité or CCE) and published by Delahaye (1989). Although the universality of $R_{K}$ has not yet been demonstrated to a level of uncertainty approaching that for the Josephson constant $K_{J}$, for dc currents in the range 10 μA to 50 μA and for ohmic contacts to the 2D electron gas with resistances $\approx 1\Omega$, Jeckelmann, Jeanneret, and Inglis (1997) have shown $R_{K}$ to be independent of device type, device material, and plateau number within their experimental relative uncertainty of about $3.5 \times 10^{-10}$. In particular, these experimenters showed that the anomalous values of $R_{K}$ observed for certain Si MOSFETs are very likely due to the resistances of the voltage contacts on the devices, and that the universal value of $R_{K}$ is found if all the criteria of the CCEM technical guidelines are met. In addition, Jeanneret et al. (1995) have shown that for a specially prepared set of GaAs/AlGaAs heterostructures of widths that varied from 10 μm to 1 mm, $R_{K}$ was independent of device width to within the $1 \times 10^{-5}$ relative uncertainty of the measurements. [Tests of the universality of $R_{K}$ have also been carried out by other researchers; see, for example,Refs. 28 to 34 of Taylor and Witt (1989) and also Delahaye, Sartrapinsky, and Witt (1989); Piquemal et al. (1991); Delahaye and Bournaud (1991); Hartland et al. (1991).]

The theory of the quantum Hall effect predicts, and the experimentally observed universality of $R_{K}$ is consistent with the prediction, that

$$R_{K} = \frac{\hbar}{e^{2}} = \frac{\mu_{0}c}{2\alpha} \approx 25.813\Omega, \tag{21}$$

where as usual $\alpha$ is the fine-structure constant. There is a vast literature on the quantum Hall effect [see, for example, the bibliography compiled by Van Degrift, Cage, and Girvin (1991) of important papers of the 1980s]. In particular, there have been many publications on the theory behind Eq. (21) and why it is believed to be an exact relation, some of which invoke rather general principles [see, for example, the books by Prange and Girvin (1990), Stone (1992), and Janßen et al. (1994), the papers for nonspecialists by Yennie (1987) and Watson (1996), and the popular article by Halperin (1986)].

In analogy with the Josephson effect, in keeping with the experimental and theoretical evidence, we assume for the purpose of the 1998 adjustment, as was assumed for the 1986 adjustment, that any correction to Eq. (21) is negligible compared to the standard uncertainty of experiments involving $R_{K}$. Currently this uncertainty is larger than $2 \times 10^{-8} R_{K}$, and it is likely to be larger than $1 \times 10^{-9} R_{K}$ for the foreseeable future. Since $\mu_{0}$ and $c$ are exact constants in the SI, this assumption and Eq. (21) imply that a measurement of $R_{K}$ in the unit $\Omega$ with a given relative standard uncertainty provides a value of $\alpha$ with the same relative standard uncertainty.

It is of interest to note that $R_{K}$, $\alpha$, and the characteristic impedance of vacuum $Z_{0} = \sqrt{\mu_{0}/\varepsilon_{0}} = \mu_{0}c \approx 377\Omega$ are related by

$$Z_{0} = 2\alpha R_{K}. \tag{22}$$
E. Conventional Josephson constant $K_{J \cdot 90}$, conventional von Klitzing constant $R_{K \cdot 90}$, and conventional electric units

It has long been recognized that the Josephson and quantum Hall effects can be used to realize accurate and reproducible representations of the (SI) volt and (SI) ohm (Taylor et al., 1967; von Klitzing et al., 1980). In order to achieve international uniformity in measurements of voltage and resistance, on 1 January 1990 the CIPM introduced new representations of the volt and the ohm for worldwide use based on these effects and conventional (i.e., adopted) values of the Josephson constant $K_J$ and von Klitzing constant $R_K$ (Quinn, 1989). These assigned exact values, denoted, respectively, by $K_{J \cdot 90}$ and $R_{K \cdot 90}$, are

$$K_{J \cdot 90} = 483 597.9 \text{ GHz/V}$$

$$R_{K \cdot 90} = 25 812.807 \text{ } \Omega.$$  

(23a)

(23b)

They were derived by the CCEM of the CIPM from an analysis of all the relevant data available by 15 June 1988 (Taylor and Witt, 1989). These data included measurements of $K_J$ and $R_K$ as well as other fundamental constants. The goal was to select conventional values of the Josephson and von Klitzing constants (within certain constraints) that were as close to their SI values as possible so that the new volt and ohm representations would closely approximate the volt and the ohm.

For the purpose of the 1998 adjustment, we interpret the CIPM's adoption of $K_{J \cdot 90}$ and $R_{K \cdot 90}$ as establishing conventional, practical units of voltage and resistance $V_{90}$ and $\Omega_{90}$ defined by

$$K_J = 483 597.9 \text{ GHz/V}$$

$$R_K = 25 812.807 \text{ } \Omega.$$  

(24a)

(24b)

(Note that $V_{90}$ and $\Omega_{90}$ are printed in italic type in recognition of the fact that they are physical quantities.) The conventional units $V_{90}$ and $\Omega_{90}$ are related to the SI units V and $\Omega$ by

$$V_{90} = \frac{K_{J \cdot 90}}{K_J} V$$

$$\Omega_{90} = \frac{R_K}{R_{K \cdot 90}} \Omega.$$  

(25a)

(25b)

which follow from Eqs. (23) and (24).

The conventional units $V_{90}$ and $\Omega_{90}$ are readily realized in the laboratory: 1 $V_{90}$ is the voltage across the terminals of an array of a large number of Josephson devices in series when the product of the total number of steps $n$ of the array and the frequency $f$ of the applied microwave radiation is exactly 483 597.9 GHz [see Eq. (18)]; and 1 $\Omega_{90}$ is exactly $i/25 812.807$ times the resistance of the $i$th quantized Hall resistance plateau [see Eq. (20)].

In practice, $V_{90}$ can be realized at the 1 V level with a relative standard uncertainty of less than $1 \times 10^{-9}$; and $\Omega_{90}$ can be realized at the 1 $\Omega$ level with a relative standard uncertainty that approaches $1 \times 10^{-9}$. Such a small uncertainty for $V_{90}$ is possible because of the development, beginning in the mid-1980s, of series arrays consisting of some 20 000 Josephson tunnel junctions on a single chip capable of generating well in excess of 10 V [see, for example, Hamilton, Burroughs, and Benz (1997); Pöpel (1992)]. The above uncertainties for $V_{90}$ and $\Omega_{90}$ have been demonstrated, for example, through comparisons carried out by the International Bureau of Weights and Measures (BIPM, Bureau International des Poids et Mesures), of the Josephson effect voltage standards and the quantum Hall effect resistance standards of the national metrology institutes of various countries with BIPM transportable versions of such standards [for Josephson effect voltage standards see, for example, Reymann et al. (1998); Quinn (1996); Witt (1995); Quinn (1994); Reymann and Witt (1993); and for quantum Hall effect resistance standards see Delahaye et al. (1997); Delahaye et al. (1996); Delahaye et al. (1995)].

Other conventional electric units follow directly from $V_{90}$ and $\Omega_{90}$. Examples are the conventional units of electric current and power, $A_{90} = V_{90}/\Omega_{90}$ and $W_{90} = V_{90}^2/\Omega_{90}$, which are related to the SI units A and W by

$$A_{90} = \frac{K_{J \cdot 90} R_{K \cdot 90}}{K_J R_K} A$$

$$W_{90} = \frac{K_{J \cdot 90}^2 R_{K \cdot 90}}{K_J^2 R_K} W.$$  

(26a)

(26b)

Equation (26b) is noteworthy because, if one assumes $K_J = 2e/h$ and $R_K = h/e^2$, then

$$W_{90} = \frac{K_{J \cdot 90}^2 R_{K \cdot 90}}{4} h.$$  

(27)

Since $K_{J \cdot 90}$ and $R_{K \cdot 90}$ have no uncertainty, an experimental determination of the unit ratio $W_{90}/W$ with a given uncertainty determines the Planck constant $h$ with the same relative uncertainty. This is the basis of the watt-balance measurements of $h$ discussed in Sec. III.G.

It is evident that for a voltage $U$,

$$U = \frac{U}{V_{90}} \frac{V_{90}}{V_{90}} = \frac{U}{V_{90}} \frac{K_{J \cdot 90}}{K_J} V.$$  

(28)

That is, the numerical value of $U$ when $U$ is expressed in the SI unit V, is equal to the numerical value of $U$ when $U$ is expressed in the conventional unit $V_{90}$ multiplied by the ratio $K_{J \cdot 90}/K_J$. Similar expressions apply to other electric quantities; those of interest here are resistance $R$, current $I$, and power $P$. To summarize,

$$U = \frac{U}{V_{90}} \frac{K_{J \cdot 90}}{K_J} V$$

$$R = \frac{R}{\Omega_{90}} \frac{R_K}{R_{K \cdot 90}} \Omega$$

$$I = \frac{I}{A_{90}} \frac{K_{J \cdot 90} R_{K \cdot 90}}{K_J R_K} A$$

$$P = \frac{P}{W_{90}} \frac{K_{J \cdot 90}^2 R_{K \cdot 90}}{K_J^2 R_K} W.$$  

(29a)

(29b)

(29c)

(29d)
Throughout the 1998 adjustment we attempt to express all electric-unit-dependent quantities in terms of conventional electric units. However, in some experiments carried out prior to 1990, an alternative value of $K_f$ was adopted to define the laboratory unit of voltage $V_{LAB}$. We denote such values by $K_{J,LAB}$ and apply appropriate factors to convert to $K_{J,90}$. Further, prior to 1990, no laboratory unit of resistance was based on the conventional value of $R_K$, but in most cases of interest the laboratory unit of resistance was calibrated using the quantum Hall effect. That is, $R_K$ is known in terms of $\Omega_{LAB}$ at the time of the experiment. On the other hand, if a laboratory’s practical units of voltage and resistance were based on artifact voltage and resistance standards such as standard cells and standard resistors with no connection to the Josephson or quantum Hall effects, then we have, for example, in analogy with Eq. (29a),

$$U = (U/V_{LAB})(V_{LAB}/V) \text{ V},$$

where in general the ratio $V_{LAB}/V$ is not well known.

F. Acceleration of free fall

The acceleration of free fall, or acceleration due to gravity $g$, is of course not really a fundamental physical constant: its fractional variation with height near the Earth’s surface is $-3 \times 10^{-7}/\text{m}$, its fractional variation from equator to pole is about 0.5%, and it can have significant fractional variations over a day at a fixed location, for example, of order $2 \times 10^{-7}$ at 40° latitude, due mostly to the varying influences of the moon and sun. For reference purposes, a conventional value called “standard acceleration of gravity” given by

$$g_0 = 9.806 65 \text{ m/s}^2$$ (30)

has been adopted internationally (BIPM, 1998).

A number of experiments relevant to the 1998 adjustment, for example the measurement of $K_J^2R_K$ using a watt balance (see Sec. III.G), require the determination of a force based on the weight of a standard of mass and hence the value of $g$ at the site of the measurement. Fortunately, significant advances in the development of highly accurate, portable, and commercially available absolute gravimeters have been made in recent years [see, for example, Niebauer et al. (1995) and Sasagawa et al. (1995)]. Such instruments allow $g$ to be determined at a given site with a sufficiently small uncertainty that lack of knowledge of $g$ is not a significant contributor to the uncertainty of any experiment of interest in the adjustment. Indeed, the two most recent international comparisons of absolute gravimeters, carried out in 1994 (ICAG94) and in 1997 (ICAG97) at the BIPM and organized by Working Group 6 of the International Gravity Commission, show that $g$ can be determined with modern absolute gravimeters with a relative standard uncertainty of the order of $4 \times 10^{-9}$ (Marson et al., 1995; Robertsson, 1999). Although this uncertainty is negligible compared to the approximate $9 \times 10^{-8}$ relative standard uncertainty of the most accurate experiment that requires knowledge of $g$, namely, the most recent measurement of $K_J^2R_K$ (see Sec. III.G.2), the uncertainty of $g$ may no longer be negligible if such experiments achieve their anticipated level of uncertainty.

III. REVIEW OF DATA

This portion of the paper reviews the experimental data relevant to the 1998 adjustment of the values of the constants and in some cases the associated theory required for their interpretation. As summarized in Appendix E, in a least-squares analysis of the fundamental constants the numerical data, both experimental and theoretical, also called observational data or input data, are expressed as functions of a set of independent variables called adjusted constants. The functions that relate the input data to the adjusted constants are called observational equations, and the least-squares procedure provides best estimated values, in the least-squares sense, of the adjusted constants. Thus the focus of this Review of Data section is the identification and discussion of the input data and observational equations of interest for the 1998 adjustment. Although not all observational equations that we use are explicitly given in the text, all are summarized in Tables XVII.A.2 and XIX.B.2 of Sec. IV.C.

A. Relative atomic masses

We consider here the relative atomic masses $A_i(X)$ (see Sec. II.C) of a number of particles and atoms that are of interest for the 1998 adjustment. In this work, the relative atomic masses of the electron $A_i(e)$, neutron $A_i(n)$, proton $A_i(p)$, deuteron $A_i(d)$, helium $A_i(h)$ (the helium h is the nucleus of the $^3\text{He}$ atom), and alpha particle $A_i(\alpha)$ are included in the set of adjusted constants. The relevant data are summarized in Tables II to V, and are discussed in the following sections.

1. Atomic mass evaluation: 1995 update

A self-consistent set of values of the relative atomic masses of the neutron and neutral atoms has been peri-