Heavy Quark and Quarkonia Transport in AdS/CFT

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• This work was done in close collaboration with Jorge Casalderrey-Solana
  – Jorge Casalderrey-Solana, DT; hep-th/0701123
  – Jorge Casalderrey-Solana, DT; hep-ph/0605199

• Work on Quarkonia
  – DT, Clint Young, Kevin Dusling, Johanna Erdmenger, Felix Rust, Matthias Gross
Observation:

There is a large momentum anisotropy:

$$v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} \approx 20\%$$

Interpretation

- The medium responds as a fluid to differences in $X$ and $Y$ pressure gradients

Hydro models “work”
Estimate of transport times with Heavy Quarks

- Put a heavy quark in this medium

\[
\delta \theta \sim \sqrt{\frac{T}{m}}
\]

- The charm quark undergoes a random walk suffering many collisions

- The relaxation time of the heavy quark is:

\[
\tau_{\text{charm}} \sim \frac{M}{T} \tau_{\text{light}}
\]

If you think you know the relaxation time you should be able to compute the charm spectrum.
Langevin description of heavy quark thermalization:

- Write down an equation of motion for the heavy quarks.

\[
\frac{dx}{dt} = \frac{p}{M} \\
\frac{dp}{dt} = -\eta_D p + \xi(t) \\
\]

Drag \hspace{1cm} \text{Random Force}

- The drag and the random force are related

\[
\langle \xi_i(t) \xi_j(t') \rangle = \frac{\kappa}{3} \delta_{ij} \delta(t - t') \hspace{1cm} \eta_D = \frac{\kappa}{2MT} \\
\]

\(\kappa = \text{Mean Squared Momentum Transfer per Time}\)

- Einstein related the diffusion coefficient to the mean squared momentum transfer

\[
D = 2T^2 / \kappa \\
\]

All parameters are related to the heavy quark diffusion coefficient or \(\kappa\)
The heavy quarks will either relax to the thermal spectrum and show the same $v_2$ as all thermal particles or not depending on the Drag/Diffusion coefficients and $p_T$. 
Summary

1. Suppression and Elliptic Flow are intimately related.

2. From the suppression pattern, we estimate that

$$D \lesssim \frac{12}{2\pi T}$$

With this diffusion coefficient, I can’t produce enough elliptic flow.
Matching Langevin to a Microscopic Theory

• Heavy Quarks are Quasi Classical

\[ \lambda_{\text{de Broglie}} \sim \frac{\hbar}{\sqrt{MT}} \ll \frac{\hbar}{T} \]

• Compare the Langevin process to the microscopic theory

<table>
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<th>Microscopic Theory</th>
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Diffusion Coefficient ↔ Electric Field Correlator
QCD and Color: Heuristic Derivation

\[ p = \sqrt{MT} \]

- Use Wong equations for a classical color charge

**Langevin**

\[
\frac{dp}{dt} = -\eta_D p + \xi(t)
\]

**Microscopic Theory**

\[
\frac{dp}{dt} = \text{Tr} [Q E(t, x)]
\]

\[
\partial_t Q = -i[Q, A_0(t, x)]
\]

Color Precession

- Then compare to the microscopic theory:

\[
\kappa = \frac{C_R}{d_A} \int dt \left\langle E^a(t, x) U_{ab}(t) E^b(0, x) \right\rangle
\]
Computing $\kappa$ – Kinetic Theory vs. Correlators

- $\kappa$ is the mean squared momentum transfer per unit time:

$$\kappa = \int_{p,q} q^2 n(p)(1 + n(p')) \left|M_{\text{glue}}\right|^2$$

- $\kappa$ is the Electric Field Correlator:

The Same Thing
Wilson Lines and the Diffusion Coefficient:

- Variations of Wilson Lines give electric field correlators

\[
\langle \frac{1}{i^2} \frac{\delta^2 W[\delta x]}{\delta x(t_1) \delta x(t_2)} \rangle \sim \langle T [ E(t_2) E(t_1) ] \rangle
\]

- Put the Wilson line along the Schwinger-Keyldish contour.

\[
\kappa = \frac{1}{3} \lim_{\omega \to 0} \int dt \ e^{+i\omega t} \left\langle \frac{1}{i^2} \frac{\delta^2 W[\delta x]}{\delta x_2(t) \delta x_1(0)} \right\rangle
\]
The AdS/CFT correspondence and the Wilson Line – Maldacena, S.Rey

Measure the diffusion of the heavy boson with the dynamics of the string
Testing the string dynamics:

\[ W[\delta x] \]

\[
\frac{1}{\langle W_C[0] \rangle} \langle W_C[\delta y] \rangle = \frac{1}{e^{iS_{cl}[0]}} e^{iS_{cl}[\delta y]}
\]
Summary of Technical Steps

- Fluctuate the end point of a string at \( r = \infty \) at very small frequencies: \( u = 1/r^2 \)

\[
S_{NG} = \frac{R^2}{2\pi \ell_s^2} \int \frac{dt \, dr}{\text{Area}} \left[ 1 - \frac{1}{2} \left( \frac{\dot{y}^2}{f} - 4fu \left( y' \right)^2 \right) \right]
\]

**Quadratic Fluctuations**

- Solve for the waves along the string in the \( AdS_5 \times S_5 \) metric
  - Impose incoming boundary conditions for retarded propagator.

- Differentiating twice with respect to \( \delta y \) yields the retarded force force correlator

\[
\langle E(t)E(0) \rangle \sim \frac{\delta^2}{\delta y(t)\delta y(0)} \langle W[\delta y] \rangle \sim \frac{\delta^2}{\delta y(t)\delta y(0)} e^{iS_{NG}[\delta y]}
\]

\( \kappa \) is the imaginary part of the retarded correlator
Results:

- Mean Squared momentum Transfer

\[ \kappa = \pi \sqrt[3]{\frac{\lambda}{\kappa}} \times \frac{T^3}{\text{Large } g^2 N_c} \]

- Answer:

\[ D = \frac{2T^2}{\kappa} \]
\[ = \frac{2}{\sqrt[3]{\lambda \pi T}} \]
QCD Guesses: Strong Coupling

- Strong Coupling: $\mathcal{N} = 4$ SUSY. $\lambda \approx 5 \leftrightarrow 20$

$$D = \frac{1}{\sqrt{g_{YM}^2 N_c}} \frac{4}{2\pi T} \quad \longrightarrow \quad D \approx \frac{1.0 \leftrightarrow 2.0}{2\pi T}$$

- Weak coupling (Aleski Vuorinnen)

2 gluons + 6 scalars + 8 fermions $\neq$ 2 gluons

$$\frac{D_{QCD}}{D_{SYM}} = \frac{6}{1 + \frac{N_f}{2N_c}} \approx 4$$

- Best guess for QCD from strong coupling

$$D \approx \frac{4.0 \leftrightarrow 8.0}{2\pi T}$$

Compare to weak coupling best guess $D \approx 6/(2\pi T)$
Heavy Quark Diffusion is Parametrically Small

\[ D = \frac{1}{\sqrt{g_{YM}^2 N_c}} \frac{4}{2\pi T} \]

\[ \frac{\eta}{e + p} = \frac{1}{4\pi T} \]
Constraint On The Heavy Quark Mass

- To treat the heavy quark as a quasi-classical quasi-particle we need

\[ \tau_R \gg \frac{\hbar}{T} \]

- Then we have

\[ \tau_R \sim \frac{M}{T} \frac{D}{D} = \frac{2}{\sqrt{\lambda \pi T}} \]

- This leads to a constraint on Mass/String Length

\[ M \gg \frac{\pi T}{2} \sqrt{\lambda} \quad \quad L \gg r_o \]

- Substituting numbers we have

\[ M \gg 1.7 \text{GeV} \left( \frac{T}{0.250 \text{GeV}} \right) \left( \frac{\alpha_{SYM} N}{1.5} \right)^{1/2} \]
Generalize to Relativistic Heavy Quarks

Before

\[ \kappa_T \]

\[ \kappa_L \]

After

\[ \kappa_T \]

\[ \kappa_L \]

- Average Energy Loss

\[
\frac{d\bar{p}}{dt} = -\eta(p)\bar{p}
\] (1)

- Transverse Momentum Broadening of a heavy quark (analagous to \( \hat{q} \))

\[ \kappa_T(v) \quad = \quad \text{Mean squared transverse momentum transfer per unit time} \]

\[ \kappa_L(v) \quad = \quad \text{Mean squared longitudinal momentum transfer per unit time} \]
Moving Heavy Quarks (HKKKY and S. Gubser)

- Turn on an electric field to accelerate the quark

- Find a quasi-steady state

- Calculate the momentum flow out of the string

\[ \frac{dp}{dt} = - \eta D \underbrace{p} \quad \text{Constant!} \]

- Drag is independent of momentum!
Fluctuating the String (J. Casalderrey, DT; S. Gubser)

\[ \sqrt{\gamma} r_o \quad \rightarrow \quad \sqrt{\gamma} \pi T \]

\[ r_o \quad \rightarrow \quad \pi T \]
Before After

- Transverse Momentum Broadening of a heavy quark (analogous to $\hat{q}$)

\[
\kappa_T(v) = \sqrt{\lambda \pi T^3} \times \sqrt{\gamma}
\]

Non-relativistic result

\[
\kappa_L(v) = \sqrt{\lambda \pi T^3} \times \gamma^{5/2} \iff \text{(Gubser)}
\]

Non-relativistic result

The strong dependence on the energy could have important phenomenological consequences and theoretical implications.
First Look at Phenom Consequences: (M. Gyulassy and Horowitz)

- Suggested looking at very high momenta quarks

\[ \frac{dp}{dt} \propto p \text{ is very weird!} \]

- Fluctuations not included

\[ \gamma_{\text{max}} = \frac{M^2}{\sqrt{\lambda T}^2} \]

Much more to think about here
Quarkonia at Weak and Strong Coupling

- Collaborators:
  - Clint Young, Kevin Dusling
  - Johanna Erdmenger, Felix Rust, Matthias Gross

Work in progress!
Motivation:

- Would be a great curve indeed!
- Bottomonium states seem to survive above $T_C$
- Want to calculate the “no-coalescence” curve at strong coupling:

  Will $J/\psi$ and $\Upsilon$ follow the flow?
The Heavy Ion Swampland
The J/ψ Puzzle

- Configuration of cccbar state
- Absorption d+Au constraint?
- Shadowing or coherence
- CGC - less charm at forward rapidity
- Comovers more mid-rapidity suppression
- Lattice & dynamical screening J/ψ not destroyed?
- Large gluon density destroys J/ψ's
- Sequential screening χC, ψ', 1st, J/ψ later
- ~40% feedown from χC, ψ' (uncertain fraction)

PHENIX J/ψ Suppression:
- Like SPS at mid-rapidity
- Stronger at forward rapidity
- With forw/mid ~0.6 saturation
- χC, ψ', centrality index

Regeneration & destruction less suppression at mid-rapidity
- Narrowing of pT & y J/ψ flow

Regeneration (in medium?)
- Large charm cross section
- Charm dE/dx & flow

Slide from Mike Leitch!
Quarkonia:

- Gluons probe the structure of the quarkonium system when:

\[
\left(\pi T\right) a_o \sim 1
\]

- For bottomonium we have:

\[
\pi T a_o = 0.7 \left(\frac{T}{0.240 \text{ GeV}}\right) \left(\frac{a_0}{1.1 \text{ GeV}^{-1}}\right)
\]

Dipole approximation seems marginal – but \((\pi T a_o)^3 \simeq 0.33\)
But lattice differently – Hatsuda, Petreczky


- Bottomonia survives above $T_C$ until $1.5T_C$ (Not totally settled.)

- No mass shift! Peter says:

$$\delta M < 50 \div 100 \text{ MeV}$$

Quarkonia are weakly interacting $\Longrightarrow$ Dipole approximation good!
Dipole Interactions:

- Dipole moment $\propto \mathcal{E}$

\[ p = \alpha \mathcal{E}^2 \]

\[ U_I = -\frac{1}{2} \alpha \mathcal{E}^2 \]

- Effective Lagrangian:
  
  - Write down the lowest dimension gauge invariant terms (Manohar '94)
  
  - higher dimension Ops suppressed by small size: $(\pi T a_o)^3$

\[ \mathcal{L} = \phi_v \partial_t \phi_v + \frac{1}{2} \alpha \mathcal{E}^2 \phi_v^\dagger \phi_v + \frac{1}{2} \alpha \mathcal{B}^2 \phi_v^\dagger \phi_v + \ldots \]

- From Gravity perspective we will write this as

\[ \mathcal{L} = \phi_v \partial_t \phi_v + c_1 \underbrace{T^{00}}_{\frac{1}{2} (\mathcal{B}^2 + \mathcal{E}^2)} + c_2 \underbrace{F^2}_{\frac{1}{2} (\mathcal{B}^2 - \mathcal{E}^2)} \]
Effective Lagrangian Virtues:

- Separates short distance physics from long

\[ \mathcal{L} = \phi_v \partial_t \phi_v + \frac{1}{2} \begin{cases} \alpha_\mathcal{E} & \text{short} \\ \mathcal{E}^2 & \text{long} \end{cases} \phi^\dagger \phi_v + \ldots \]

- Polarizability “\(\alpha_E\)” is short distance physics
  - Not really something for AdS/CFT.

\[ \alpha_\mathcal{E} = \frac{28}{3N^2} \pi a_o^3 \quad \iff \quad (\text{Peskin '80}) \]
Perturbative Mass Shift:

- Leading order perturbation theory

\[
\delta M = \langle H_I \rangle \\
= -\frac{1}{2} \alpha \langle \mathcal{E}^2 \rangle \\
\sim a_B^3/N^2 \\
\sim N^2 T^4
\]

- Numerically we have

\[
\delta M = \frac{14}{45} T(\pi T a_0)^3 \\
\approx 22.5 \text{ MeV} \left[ \frac{T}{0.250} \right] \left[ \frac{(\pi T a_0)^3}{0.3} \right]
\]

- Mass shift is finite at large \( N_c \)

Mass shift is small enough to be consistent with Lattice.
Calculation of Drag and Diffusion

- $\kappa$ is the mean squared momentum transfer per unit time

$$\kappa = \int dt \langle \mathcal{F}(t, \mathbf{x}) \mathcal{F}(0, \mathbf{x}) \rangle$$

- For the force on the dipole we have

$$\mathcal{F} = -\nabla U = \frac{1}{2} \alpha \varepsilon \nabla^{2}$$

- Then the force force correlator is

$$\kappa = \alpha^{2} \varepsilon \times \frac{1}{4} \int dt \left\langle \nabla^{2}E(t, \mathbf{x}) \nabla^{2}E(t, \mathbf{x}) \right\rangle$$

Computable in AdS
Computing $\kappa$ – Kinetic Theory vs. Correlators

- $\kappa$ is the mean squared momentum transfer per unit time:

  \[
  \kappa = \int_{\mathbf{p},\mathbf{q}} \mathbf{q}^2 n(p)(1+n(p')) \left| M_{\text{glue}} \right|^2
  \]

  Then

  \[
  \left| M_{\text{glue}} \right|^2 \propto \alpha_E^2 \omega^4
  \]

- $\kappa$ is the $\alpha_E \nabla \mathcal{E}^2$ correlator:

The Same Thing
Perturbation Theory Results and AdS/CFT Estimates

- The mean squared momentum per time

\[ \kappa = \alpha_e^2 \times \frac{1}{4} \int dt \left\langle \nabla \mathcal{E}^2(t, x) \nabla \mathcal{E}^2(t, x) \right\rangle \]

\[ = \alpha_e^2 \times \frac{64\pi^5}{45} N^2 T^9 \left( \frac{a_0^3}{N^2} \right)^2 \]

- In terms of the Bohr Radius:

\[ \kappa = \frac{41.29}{N^2} \pi T^3 (\pi Ta_0)^6 \]

\[ = \frac{(300 \text{ MeV})^2}{\text{fm}} \left[ \frac{T}{250 \text{ MeV}} \right]^3 \left[ \frac{(\pi Ta_0)^3}{0.3} \right]^2 \]

- Result does involve explicit coupling constant (like pressure)
- Suppressed by \( N^2 \)
- Coefficient is large because the phase space for \( \sigma \propto \omega^4 \) is large
Relation to the mass shift

\[ \kappa = \frac{(300 \text{ MeV})^2}{\text{fm}} \left[ \frac{(\delta M)}{22 \text{ MeV}} \right]^2 \left[ \frac{T}{250 \text{ MeV}} \right] \]

The mass shift needs to be very small in order to get modest (not huge) momentum relaxation rates
AdS/CFT and Mesons

- Insert a D7 brane into $AdS_5 \times S_5$. Dual to $\mathcal{N} = 4$ SYM + Heavy $\mathcal{N} = 2$ Quarks. (Karch, Katz)

$$S_{D7} = T_{D7} \int d^8 \sigma \sqrt{-\det G_{MN} \frac{\partial X^M}{\partial \sigma^a} \frac{\partial X^N}{\partial \sigma^b}}$$
D7 Soup

\[ r = L \]

\[ r \sim M_Q \]

\[ r = 0 \]
D7 Soup

\[ r = L \]

\[ r = M_j L^2 \]

\[ r = 0 \]
The excitations of the D7 brane are mesons; can compute the mass

- The masses are:

\[ M_J = \frac{2\pi M_Q}{\sqrt{\lambda}} \cdot 2\sqrt{2} \]

- Meson mass is less than the quark mass. Relativistic Bound state!

Binding Energy \( \gg M_Q \)
Interactions with the Medium: $M_J \gg T$

- The interactions are summarized by the effective Lagrangian

$$\mathcal{L} = \phi_v \partial_t \phi_v + c_T T^{00} + c_F O_{F^2}$$

\begin{align*}
\frac{1}{2} (B^2 + \mathcal{E}^2) + \ldots & \\
\frac{1}{2} (B^2 - \mathcal{E}^2) + \ldots
\end{align*}

- These are:
  - The lowest dimension ops which are $SU(4)$ singlets
  - These operators have protected anomalous dimension
D7 Soup

\[ r = L \]
\[ r = M \]
\[ r_0 = \pi TL^2 \]
\[ r = 0 \]
The excitations of the D7 brane are meson; one can compute the mass

The masses are:

\[
M = M_J + \#T \left( \frac{T}{M_J} \right)^3
\]

\[
= M_J + c_T \times \left\langle T^{00} \right\rangle
\]

\[
\sim \frac{1}{M_J^3 N^2} \sim N^2 T^4
\]

Still working on #.

Basically involves replacement

\[
\alpha \varepsilon \propto a_0^3 \frac{1}{N^2} \quad \iff \quad c_T = \frac{1}{M_J^3} \frac{1}{N^2}
\]
Putting it all together:

- The interactions are summarized by the effective Lagrangian

\[ \mathcal{L} = \phi_v \partial_t \phi_v + c_T T^{00} + c_F \mathcal{O}_F^2 \]

- The Force is the gradient of the Potential

\[ \mathcal{F} = \nabla U \quad U = c_T T^{00} + c_F \mathcal{O}_F^2 \]

- So

\[ \kappa = \int dt \langle \mathcal{F}(t) \mathcal{F}(0) \rangle \]

\[ \sim c_T^2 \int dt \langle T^{00}(t) T^{00}(0) \rangle \]

\[ \sim \# \times \frac{1}{M_6^6 N^4} \times N^2 T^9 \]
Summary

- For the Strong Coupling we have

\[ \kappa = \sqrt{\frac{41.29}{N^2}} \pi T^3 \times \left[ \frac{\pi T}{M_J} \right]^6 \]

- Drag and diffusion Heavy Meson – Perturbative

\[ \kappa = \frac{41.29}{N^2} \pi T^3 \times [\pi T a_0]^6 \]

- For Both Weak and Strong Coupling we have

\[ \kappa = \left( \delta M \right)^2 \pi T \frac{1280}{3N^2} \times \# \]

\[ \kappa = \frac{(300 \text{ MeV})^2}{\text{fm}} \left[ \frac{\delta M}{22 \text{ MeV}} \right]^2 \left[ \frac{T}{250 \text{ MeV}} \right] \]
Summary

- AdS/CFT is a useful foil to perturbation theory
- It's a wild theory!
- Many Questions (suitably posed) can be answered.
- My goal: find something computable in AdS/CFT in wild contradiction with data.
  - We haven't found it yet.