AdS/CFT Day at Columbia, October 26, 2007

Conformal symmetry breaking in QCD and implications for hot quark-gluon matter

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Bulk viscosity of QCD matter:

the tale of “the least studied transport coefficient”

Based on:

DK, K. Tuchin, arXiv:0705.4280 [hep-ph]

F. Karsch, DK, K. Tuchin, to appear

Shear and bulk viscosities: the definitions

The energy-momentum tensor:

$$\theta_{ij} = P_{eq}(\epsilon)\delta_{ij} - \eta \left(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u_k\right) - \zeta\, \delta_{ij}\vec{\nabla}\cdot\vec{u}$$

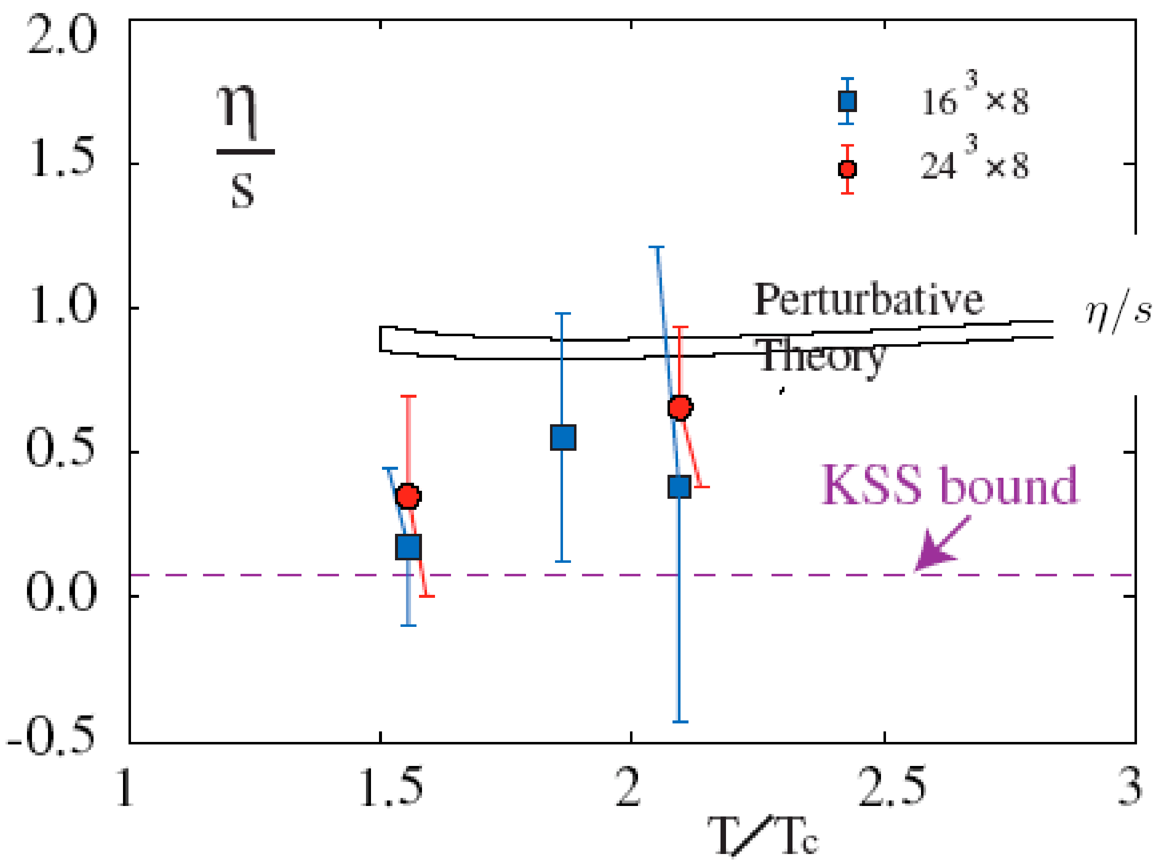
shear viscosity

bulk viscosity

Shear viscosity has attracted a lot of attention:

A.Nakamura and S.Sakai, hep-lat/0406009;

Recent work:
H.Meyer, 0704.1801



$$\eta/s = \begin{cases} 0.134(33) & (T = 1.65T_c) \\ 0.102(56) & (T = 1.24T_c) \end{cases}$$

Perfect liquid

Kovtun - Son - Starinets bound:
strongly coupled SUSY QCD = classical supergravity

Kubo's formula:

$$\eta(\omega)\left(\delta_{il}\delta_{km}+\delta_{im}\delta_{kl}-\frac{2}{3}\delta_{ik}\delta_{lm}\right)+\zeta(\omega)\delta_{ik}\delta_{lm}$$
$$=\frac{1}{\omega}\lim_{\mathbf{k}\to\mathbf{0}}\int d^3x\int_0^\infty dt\, e^{i(\omega t-\mathbf{kr})}\langle[\theta_{ik}(t,\mathbf{r}),\theta_{lm}(0)]\rangle$$

Bulk viscosity is defined as the static limit of the correlation function:

$$\zeta=\frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega}\int_0^\infty dt\int d^3r\, e^{i\omega t}\langle[\theta_{ii}(x),\theta_{kk}(0)]\rangle$$

Kubo's formula for bulk viscosity can be written down in the form involving Lorentz-invariant operators:

$$\left\langle \left[\int d^3x\, \theta_{00}(x), \mathfrak{D} \right] \right\rangle_{\text{eq}} = \langle [H, \mathfrak{D}] \rangle_{\text{eq}} = i \left\langle \frac{\partial \mathfrak{D}}{\partial t} \right\rangle_{\text{eq}} = 0$$

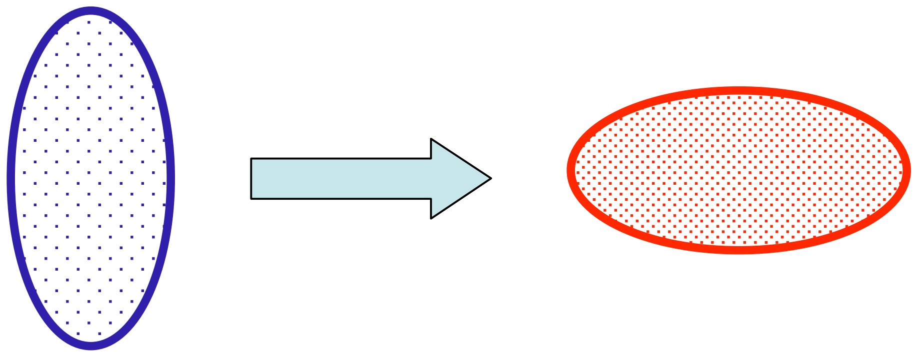
Since θ_{00}'s commute, we get

$$\zeta = \frac{1}{9} \lim_{\omega \to 0} \frac{1}{\omega} \int_0^\infty dt \int d^3r\, e^{i\omega t} \langle [\theta_\mu^\mu(x), \theta_\mu^\mu(0)] \rangle \,.$$

Bulk viscosity is determined by the correlation function of the trace of the energy-momentum tensor

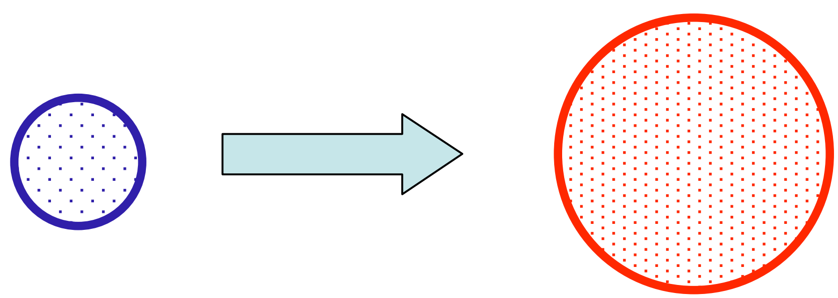
Physical picture:

Shear viscosity: how much entropy is produced by transformation of shape at constant volume



Generated by translations

Bulk viscosity: how much entropy is produced by transformation of volume at constant shape



Generated by dilatations

Scale invariance in field theory

Scale transformation

$$x \to \lambda x$$

Scale current's divergence is equal to the trace of the energy-momentum tensor

$$\partial_\mu s^\mu = \theta^\mu_\mu$$

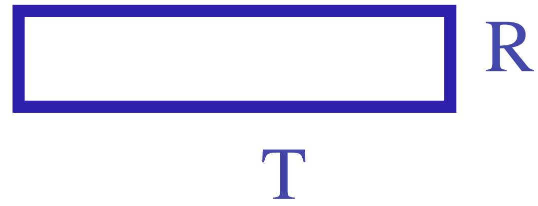
In scale-invariant theories (such as $N = 4$ SUSY YM), $\theta^\mu_\mu = 0$

Since bulk viscosity is related to the correlation function of θ^μ_μ, in scale-invariant theories bulk viscosity vanishes.

How big is the bulk viscosity of QCD plasma?

Scale invariance and confinement

Consider a rectangular Wilson loop:



$$W(C) = \exp\left(ig\int_C A_\mu dx^\mu\right)$$

It is related to the potential $V(R)$ acting between the charges Q and $\bar{Q}$:

$$W(C) \to \exp\left(-TV(R)\right)$$

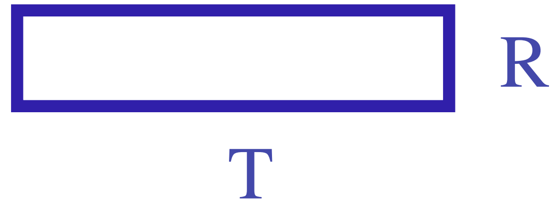
Scale transformation: $T \to \lambda T$; $R \to \lambda R$;

the only solution: Coulomb potential

$$V(R) \sim \frac{1}{R}$$

Scale invariance and confinement

$$W(C) \to \exp\left(-TV(R)\right)$$



Running coupling and especially confinement

(area law for the Wilson loop)

e.g. the potential

$$V(R) = -\frac{4}{3}\frac{\alpha_s(R)}{R} + \sigma R$$

explicitly break scale invariance

Scale anomaly in QCD

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_f \bar{q}^a_f (i\gamma_\mu D_\mu - m_f) q^a_f;$$

Classical scale invariance is broken by quantum effects:

scale anomaly

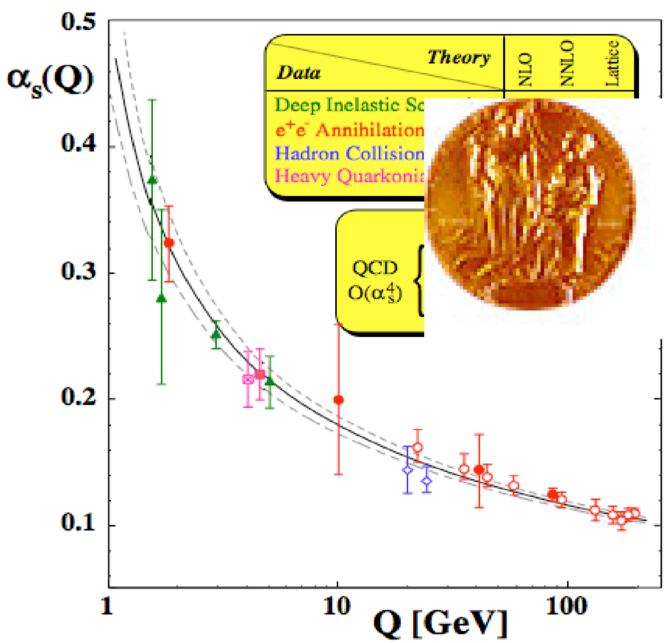
$$\theta^\mu_\mu = \frac{\beta(g)}{2g} G^{\alpha\beta a} G^a_{\alpha\beta} + \sum_q m_q \bar{q} q$$

trace of the energy-momentum tensor

"beta-function"; describes the dependence of coupling on momentum $\mu \frac{dg(\mu)}{d\mu} = \beta(g)$

Hadrons get masses ⟺ coupling runs with the distance

Asymptotic Freedom



At short distances,
the strong force becomes weak
(anti-screening) -
one can access the "asymptotically free" regime in hard processes

and in super-dense matter
(inter-particle distances ~ 1/T)

$$\alpha_s(Q) \simeq \frac{4\pi}{b \ln(Q^2/\Lambda^2)}$$

number of colors

number of flavors

$$b = (11 N_c - 2 N_f)/3$$

Renormalization group: running with the field strength

RG constraints the form of the effective action:

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4\bar{g}^2(t)}\, G^2, \quad t \equiv \ln\left(\frac{G^2}{\Lambda^4}\right)$$

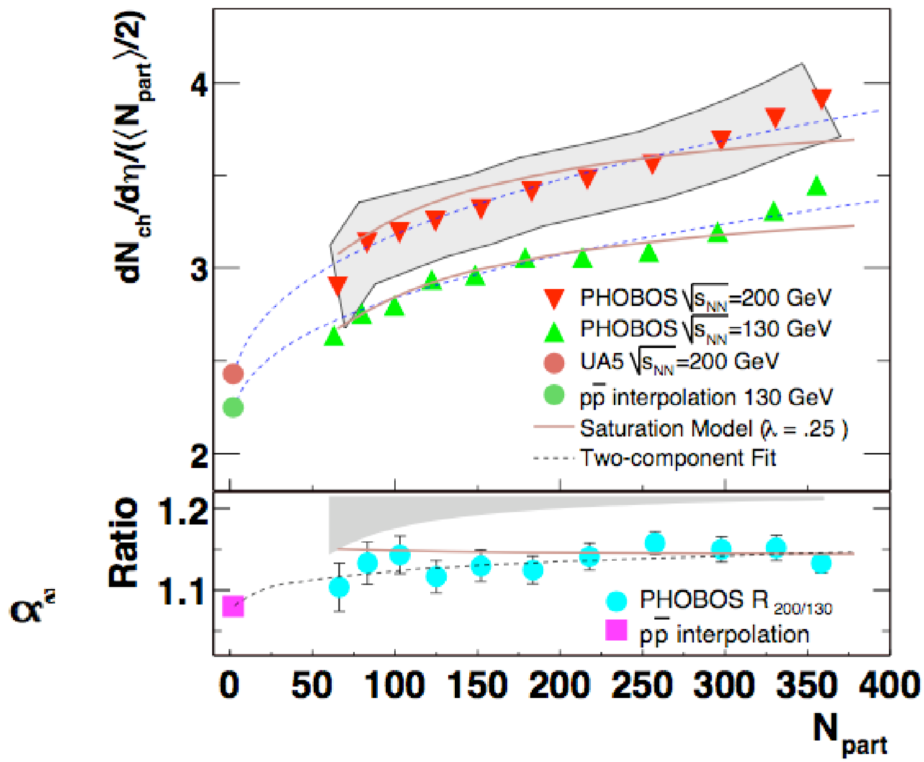
the coupling is defined through

$$t = \int_g^{\bar{g}(t)} \frac{dg}{\beta(g)};$$

At large t (strong color field),

$$\frac{1}{\bar{g}^2(t)} \sim t + \ldots \quad \text{and} \quad \mathcal{L}_{\text{eff}} \sim G^2 \, \ln\left(\frac{G^2}{\Lambda^4}\right)$$

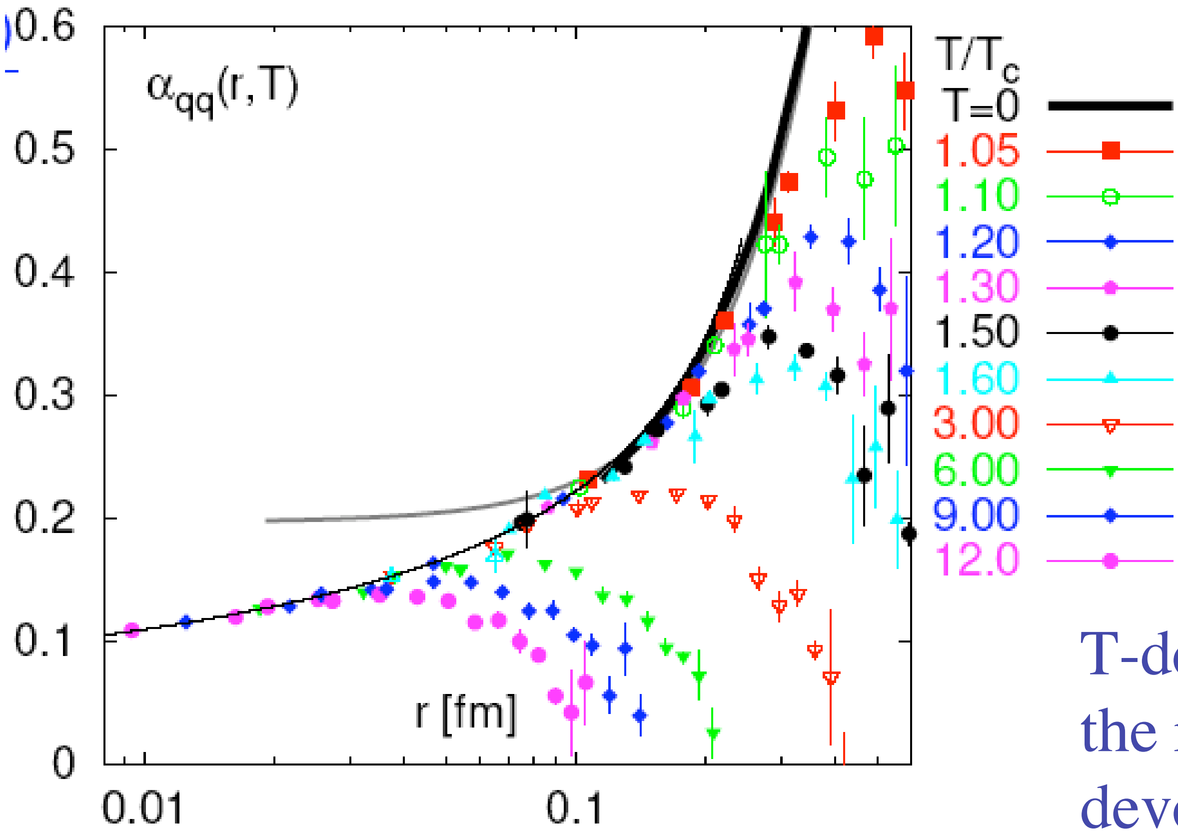
$$\frac{1}{N_{part}} \frac{dN}{d\eta} \sim \frac{1}{\alpha_s(Q_s^2)}$$



KLN

Running coupling essential for understanding hadron multiplicities

Running coupling in QGP



Strong force is screened by the presence of thermal gluons and quarks

T-dependence of the running coupling develops in the non-perturbative region at $T < 3\ T_c$; $\Delta E/T > 1$ - “cold” plasma

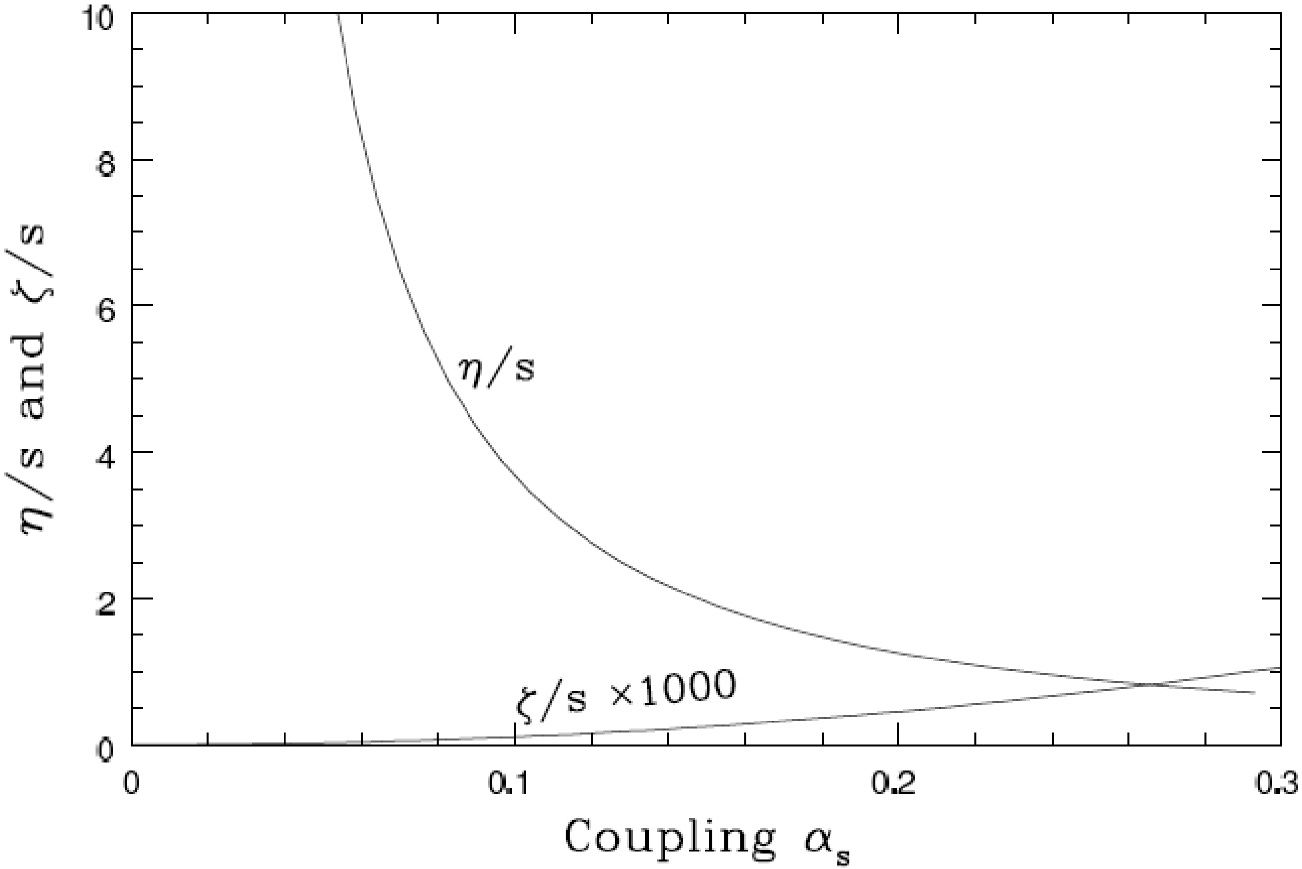
F.Karsch et al

Running coupling (and perhaps “remnants of confinement”) seen in the lattice data indicate:

Scale invariance in the quark-gluon plasma is at best approximate

What does it mean for bulk viscosity?

Perturbation theory: bulk viscosity is negligibly small



$\zeta/\eta < 10^{-3}$

$$\zeta \sim \frac{\alpha_s^2 T^3}{\log[1/\alpha_s]} \quad (m_0 \ll \alpha_s T)$$

P.Arnold, C.Dogan, G.Moore, hep-ph/0608012

In perturbation theory, shear viscosity is “large”:

$$\frac{\eta}{s} \sim \frac{1}{\alpha_s^2}$$

and bulk viscosity is “small”:

$$\frac{\zeta}{s} \sim \alpha_s^2$$

At strong coupling, η is apparently small;

can ζ get large?

Can we say anything about non-perturbative effects?

At zero temperature, broken scale invariance leads to a chain of low-energy theorems for the correlation functions of $\partial^\mu s_\mu = \theta^\mu_\mu$

Novikov, Shifman, Vainshtein, Zakharov '81

Elegant geometrical interpretation - classical theory in a curved gravitational background - Einstein-Hilbert action, etc

Migdal, Shifman '82; DK, Levin, Tuchin '04

These theorems have been generalized to finite T:

$$G^E(0,\vec{0}) = \int d^4x \, \langle T\theta(x), \theta(0)\rangle = \left(T\frac{\partial}{\partial T} - 4\right) \langle\theta\rangle_T$$

Ellis, Kapusta, Tang '98

Sketch of the derivation

Consider an operator with a canonical dimension d:

$$\langle O \rangle \;\sim\; \left[M_0 \, e^{-\frac{8\pi}{b\, g^2(\mu)}} \right]^d$$

The dependence of QCD Lagrangian on the coupling:

$$\mathcal{L}_{\rm QCD} \;=\; (-1/4g^2)\,\tilde{F}^{a\mu\nu}\,\tilde{F}^a_{\mu\nu} \qquad \tilde{F} \;=\; g F$$

Write down an expectation value for O as
a functional integral and differentiate w.r.t. $1/4g^2$:

$$i \lim_{q\to 0} \int dx\, e^{i\,q\,x}\, \langle 0|T\{\, O(x)\,,\, \frac{\beta(\alpha_s)}{4\,\alpha_s}\, F^2(0)\,\}|0\rangle_{\rm connected} = \langle O \rangle\, (-4)$$

Repeat n times - get n-point correlation functions

An exact sum rule for bulk viscosity

Basing on LET's and Kubo's formula, we derive
an exact sum rule for the spectral density:

$$2\int_0^\infty \frac{\rho(u,\vec{0})}{u}\,du = -\left(4 - T\frac{\partial}{\partial T}\right)\langle\theta\rangle_T = T^5\frac{\partial}{\partial T}\frac{(\mathcal{E}-3P)_{\rm LAT}}{T^4} + 16|\epsilon_v|$$

Using ansatz we get

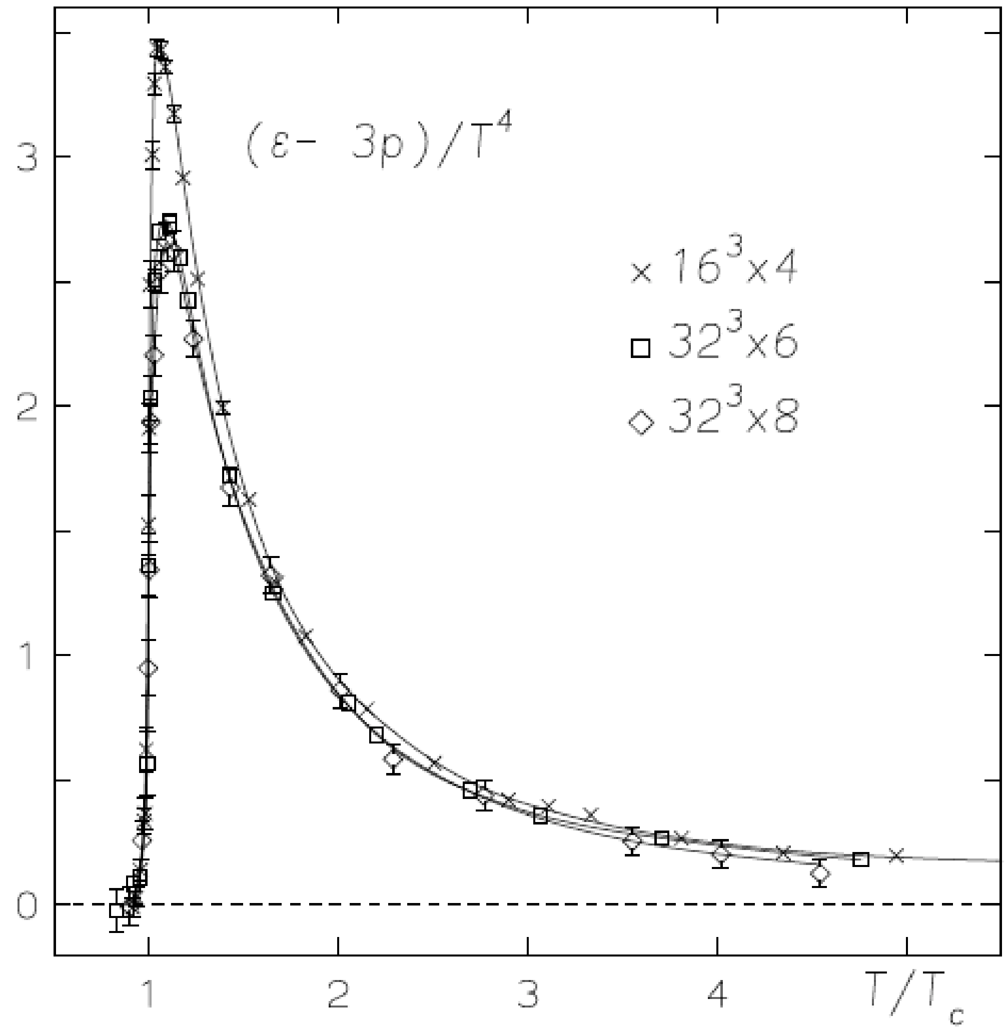
$$\frac{\rho(\omega,\vec{0})}{\omega} = \frac{9\,\zeta}{\pi}\frac{\omega_0^2}{\omega_0^2+\omega^2}$$

$$\zeta = \frac{1}{9\,\omega_0}\left\{T^5\frac{\partial}{\partial T}\frac{(\mathcal{E}-3P)_{\rm LAT}}{T^4} + 16|\epsilon_v|\right\}$$

$\omega_0 \approx (T/T_c)\ 1.4\ \mathrm{GeV}$

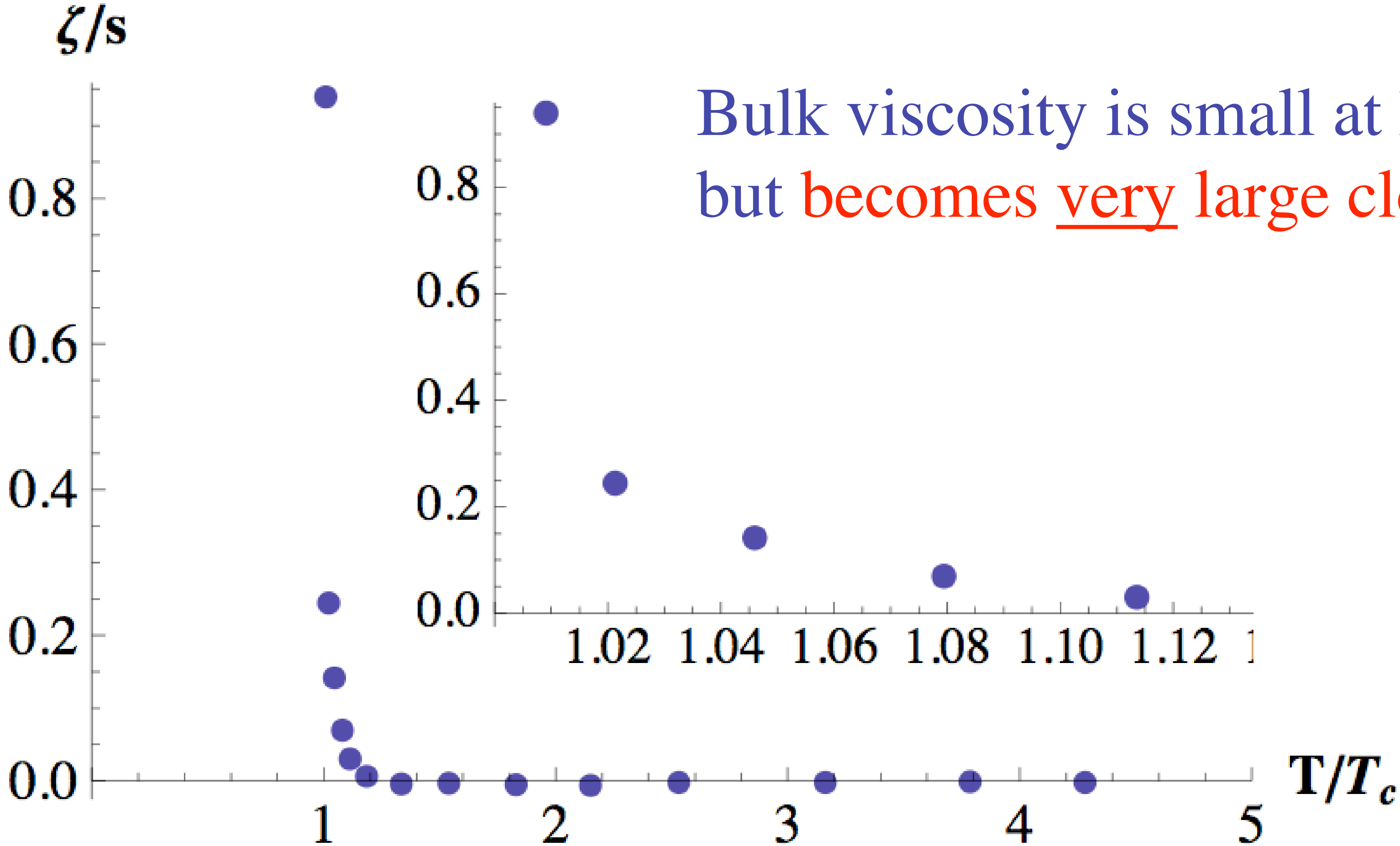
DK, K.Tuchin, arXiv:0705.4280 [hep-ph]

SU(3), pure gauge



Use the lattice data from G.Boyd, J.Engels, F.Karsch, E.Laermann, C.Legeland, M.Lutgeimer, B.Petersson, hep-lat/9602007

The result



Bulk viscosity is small at high T, but becomes <u>very</u> large close to T_c

DK, K.Tuchin, arXiv:0705.4280 [hep-ph]

Condensed matter analogies?

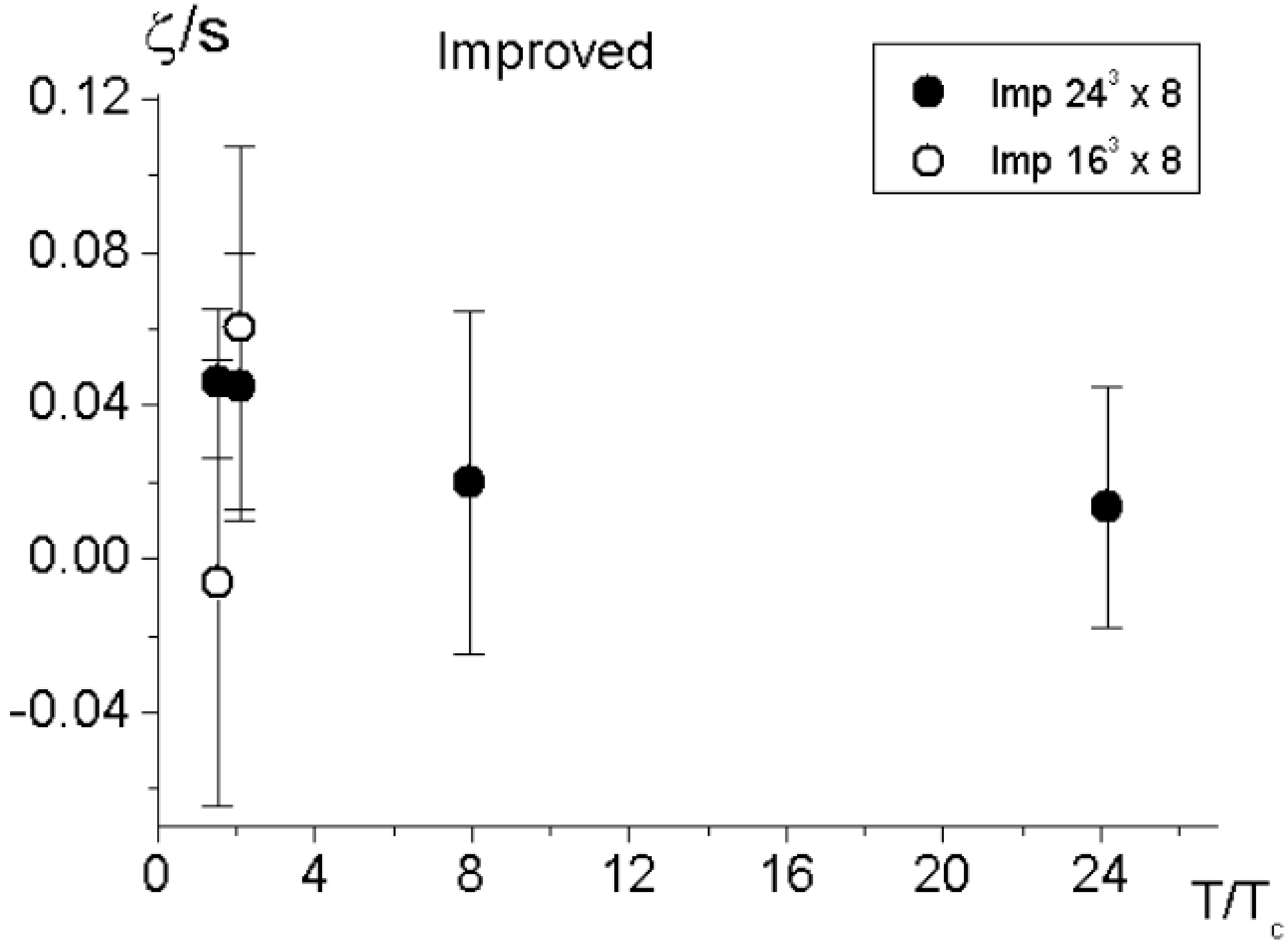
Example: ^{3}He near the critical point

at $(T-T_c)/T_c = 10^{-4}$ on the critical isochore,

shear viscosity is η=17 10^{-6} Poise
whereas bulk viscosity is ζ=50 Poise

The ratio ζ/η is in excess of a million

Kogan, A.B. and Meyer, H. (1998). Sound Propagation in ^{3}He and ^{4}He Above the Liquid–Vapor Critical Point. J. Low Temp. Phys. 110, 899.



Recently it has been claimed that the

bulk viscosity is large near the critical temperature and decreases rapidly with T[10]. Our results do not contradict this. In the case of the standard action, the bulk viscosities still have large errors that they cannot be determined.

S.Sakai, A.Nakamura, arXiv:0710.3625[hep-lat], Oct 19, 2007

MIT-CTP 3881

A calculation of the bulk viscosity in SU(3) gluodynamics

Harvey B. Meyer*
Center for Theoretical Physics
Massachusetts Institute of Technology
Cambridge, MA 02139, U.S.A.
(Dated: October 19, 2007)

We perform a lattice Monte-Carlo calculation of the trace-anomaly two-point function at finite temperature in the SU(3) gauge theory. We obtain the long-distance properties of the correlator in the continuum limit and extract the bulk viscosity ζ via a Kubo formula. Unlike the tensor correlator relevant to the shear viscosity, the scalar correlator depends strongly on temperature. If s is the entropy density, we find that ζ/s becomes rapidly small at high T, $\zeta/s < 0.15$ at $1.65T_c$ and $\zeta/s < 0.015$ at $3.2T_c$. However ζ/s rises dramatically just above T_c, with $0.5 < \zeta/s < 2.0$ at $1.02T_c$.

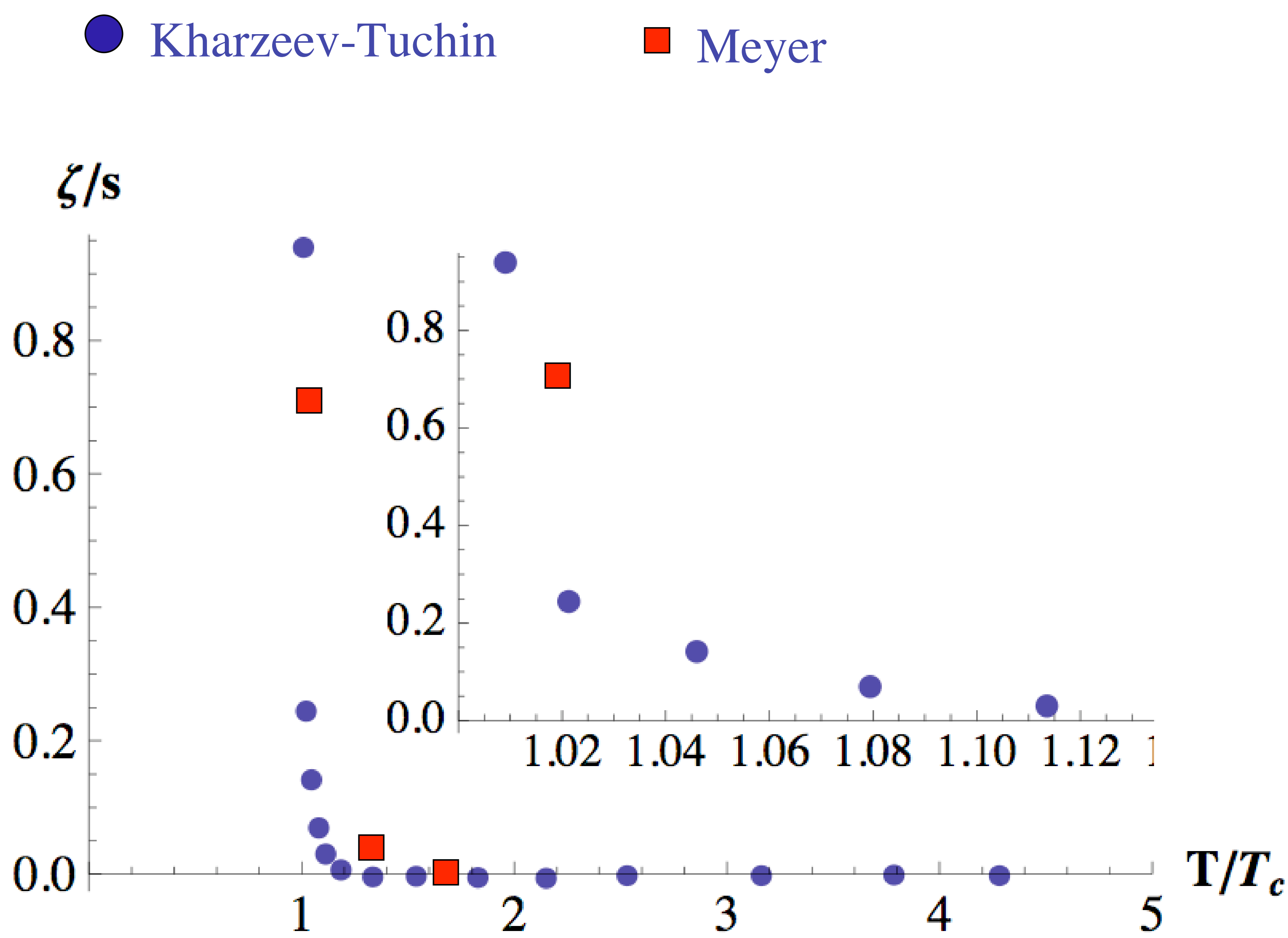
However ζ/s rises dramatically just above T_c, with $0.5 < \zeta/s < 2.0$ at $1.02T_c$.

H.Meyer, arXiv:0710.3717[hep-lat], Oct 19, 2007

$$\zeta/s = \begin{cases} 0.008(7)\begin{bmatrix} 0.15 \\ 0 \end{bmatrix} & (T = 1.65T_c,\ LT = \frac{16}{3}) \\ 0.065(17)\begin{bmatrix} 0.37 \\ 0.01 \end{bmatrix} & (T = 1.24T_c,\ LT = \frac{8}{3}). \end{cases}$$

$$\zeta/s = 0.73(3)\begin{bmatrix} 2.0 \\ 0.5 \end{bmatrix} \qquad (T = 1.02T_c,\ LT = 3).$$

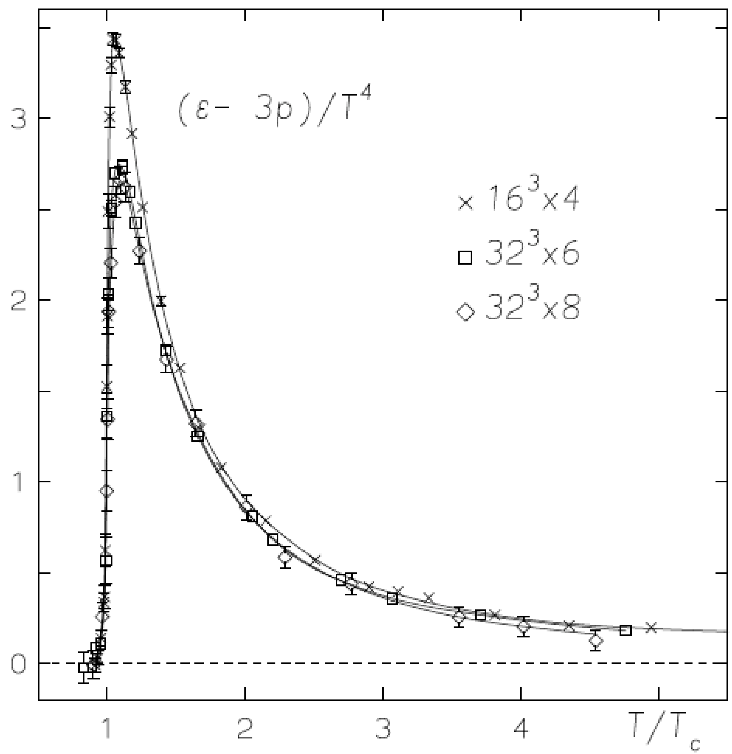
H.Meyer, arXiv:0710.3717[hep-lat], Oct 19, 2007


Kharzeev-Tuchin
Meyer
ζ/s
0.8
0.6
0.4
0.2
0.0
1
2
3
4
5
T/T_c
0.8
0.6
0.4
0.2
0.0
1.02
1.04
1.06
1.08
1.10
1.12

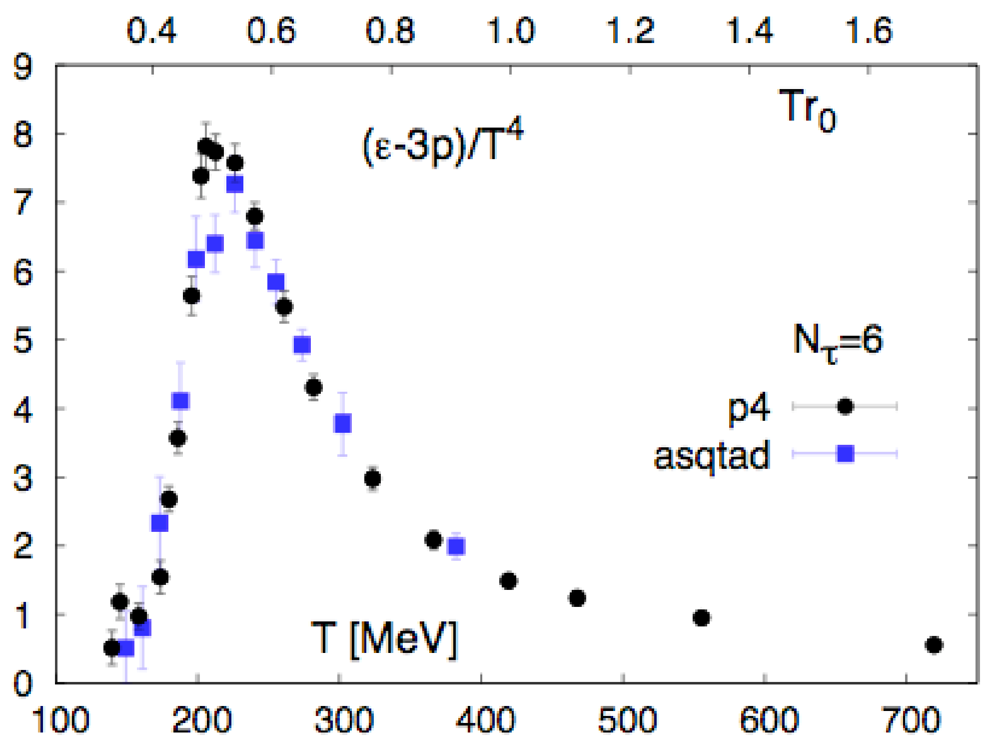
Bulk viscosity in full QCD

Qualitatively similar results:

F.Karsch, DK, K.Tuchin, to appear



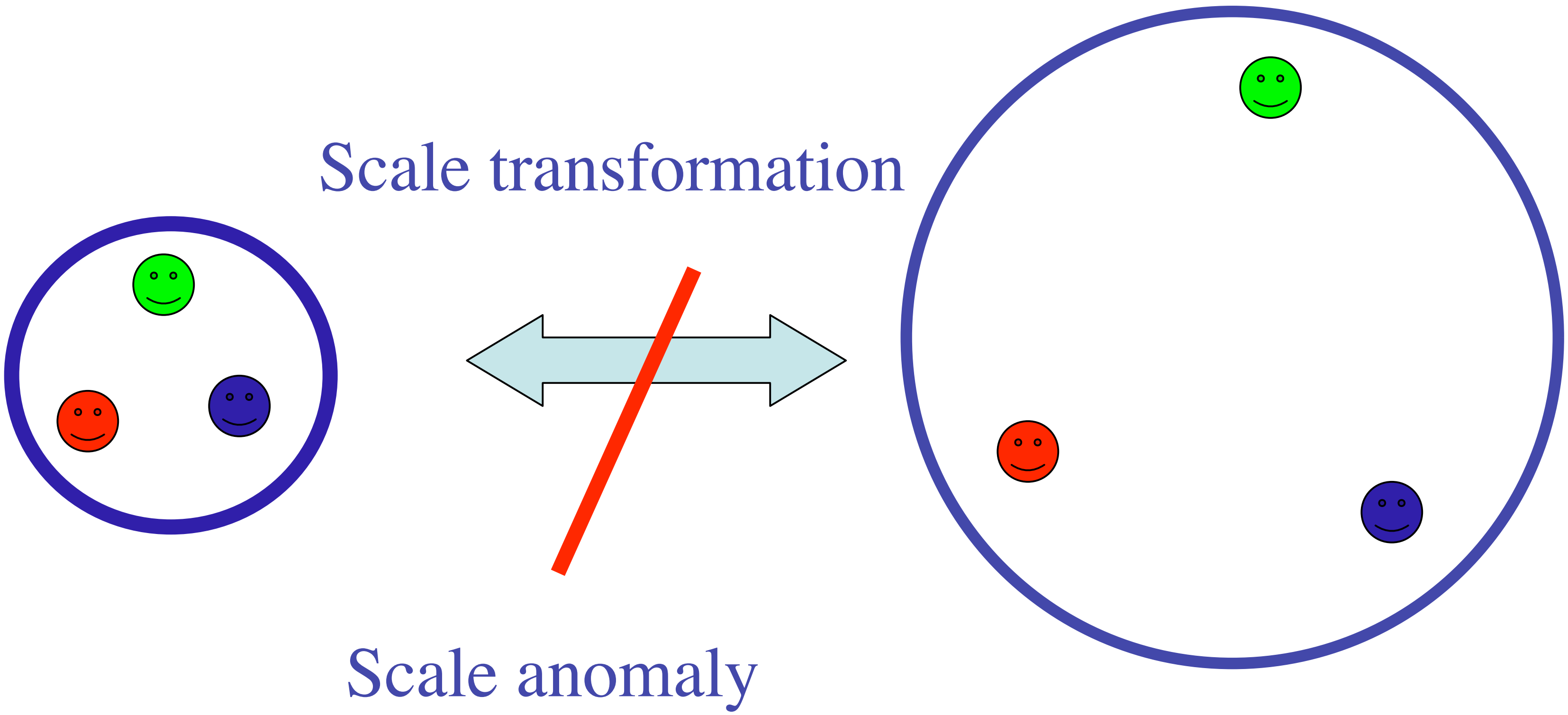
SU(3), pure gauge



QCD, 2+1 quark flavors (pion mass 220 MeV)

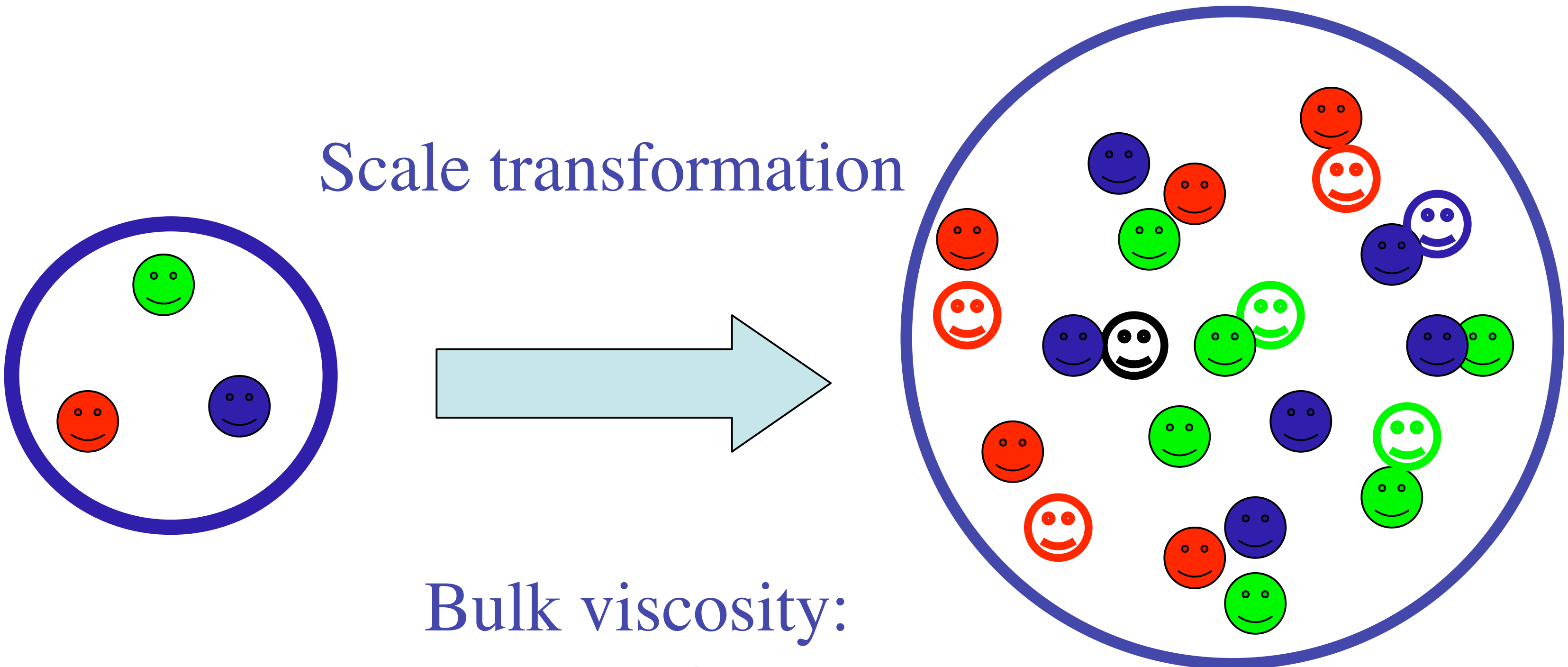
BNL-Columbia-RBRC-Bielefeld
arXiv:0710.0354

Bulk viscosity and the mechanism of hadronization



What is the meaning of the bulk viscosity growth?

Bulk viscosity and the mechanism of hadronization

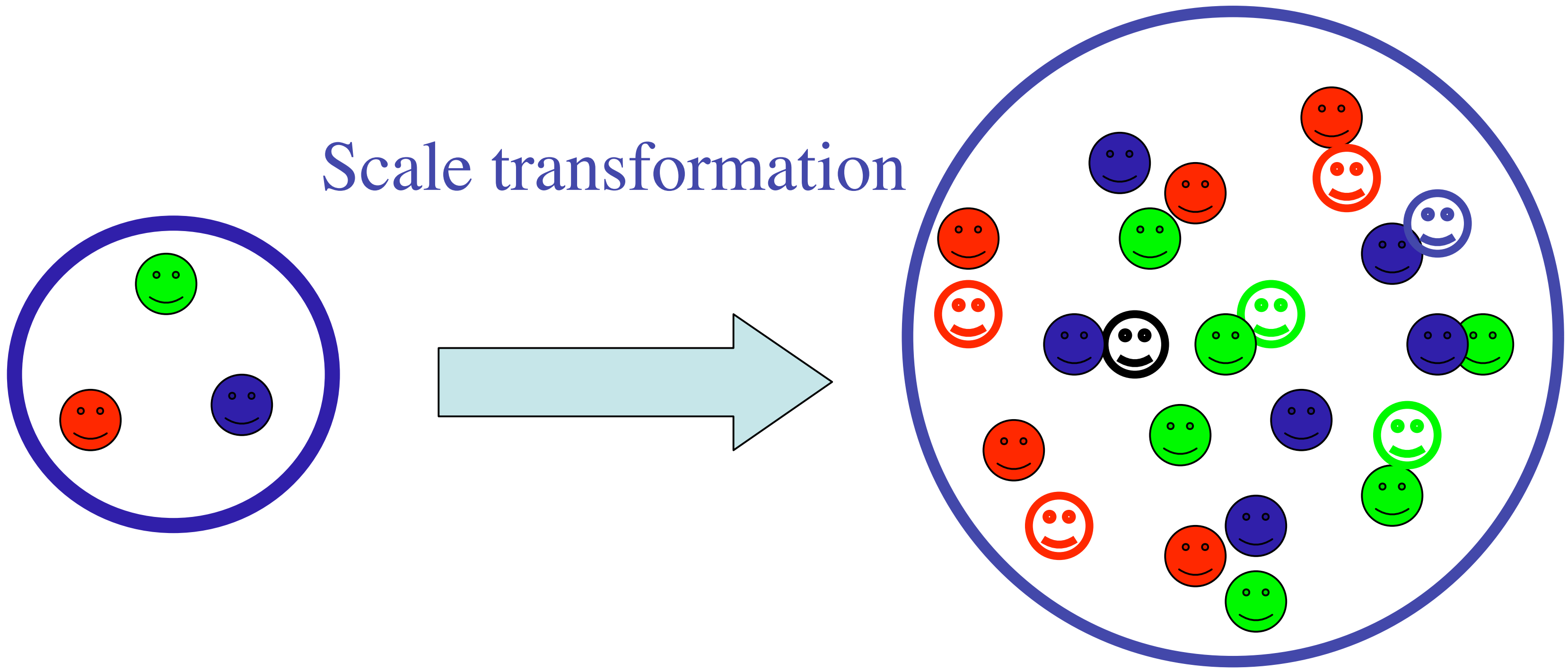


Scale transformation

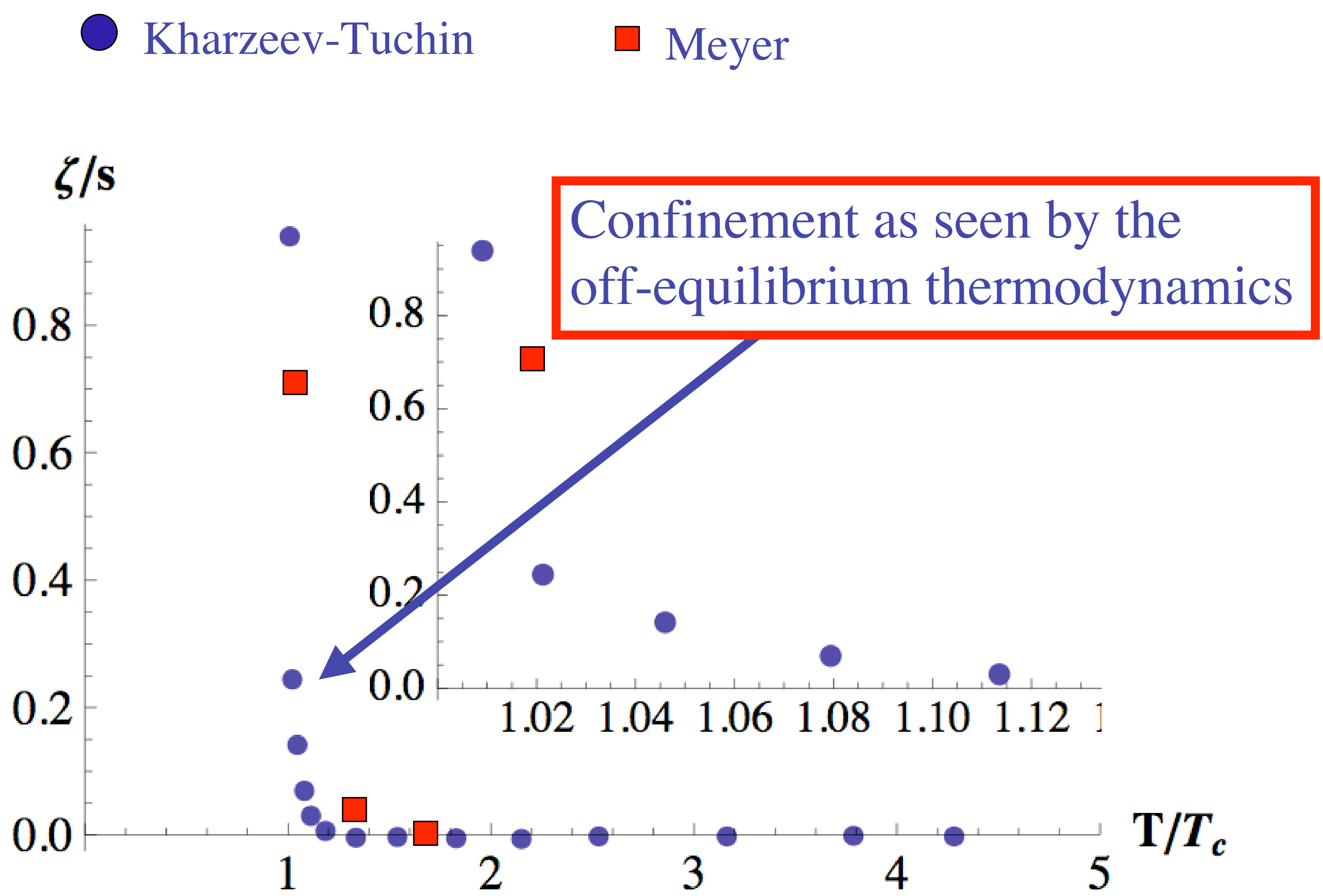
Bulk viscosity:
growth of entropy,
particle production

Bulk viscosity growth = soft statistical hadronization (?)

Bulk viscosity and the mechanism of hadronization



Not a recombination of pre-existing quarks -
Bulk viscosity saves the 2nd law of thermodynamics
in the process of hadronization


Kharzeev-Tuchin
Meyer
ζ/s
0.8
0.6
0.4
0.2
0.0
1
2
3
4
5
T/Tc
0.8
0.6
0.4
0.2
0.0
1.02
1.04
1.06
1.08
1.10
1.12
Confinement as seen by the
off-equilibrium thermodynamics

Summary

1. Bulk viscosity is small away from T_c - approximately scale-invariant dynamics, "perfect liquid"

2. Bulk viscosity grows dramatically (3 orders!) close to the critical temperature (most likely, a peak at T_c): by far, the dominant viscous effect at this temperature

3. This suggests a new scenario for soft statistical hadronization

4. Understanding the associated "microscopic" dynamics is crucial for understanding hadronization and confinement

Need to devise the methods of experimental study